Computer Optics Vol. 1, No. 1, pp. 25–28, 1989
Printed in Great Britain

0955–355X/89 $10.00 + 0.00

SYNTHESIS OF OPTICAL ANTENNAE

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Abstract—The concepts of optical antenna and optical antenna lattice are introduced. For light possessing arbitrary coherence properties an expression has been obtained for the directivity diagram in the Fresnel diffraction zone. Artificial optical elements designed by computer are suggested for generating the desired field in the optical antenna field of vision. A kinoform element has been manufactured providing the specified number of directivity diagram lobes.

Antenna design theory in radio and HF bands has been treated in detail in a series of monographs [1, 2]. Optical antennae are devices used to radiate and detect electromagnetic radiation in the optical region satisfying given directivity requirements, and enabling one to solve problems in locational and navigational technology. An attempt has been made [3] to formally transfer the theory of radio antennae to the optical range. But optical antennae have a series of distinctive features:

(1) The operating region of greatest interest for optical antennae is the zone of Fresnel diffraction, which is only a transitional zone for radio antennae.
(2) The antenna aperture field is, in general, not strictly monochromatic or spatially coherent, the angular resolution in an optical antenna being related to the spatial coherence of the source radiation.
(3) Classical optical elements like lenses, mirrors or diaphragms have only a limited capability of varying the aperture fields to obtain requisite directivity diagrams, unlike the arbitrarily controllable antenna arrays employed in radio technology.

In the present work we introduce the concept of directivity diagrams in the Fresnel diffraction zone for radiation with arbitrary degrees of coherence. In order to generate the requisite fields at the aperture we propose the use of artificial optical elements synthesized by means of a computer [4].

Suppose a flat aperture Λ of an optical antenna contains an optical element L with complex transmission function $T(\vec{u}, \nu)$ (at given frequency ν), illuminated by an extended source of light σ (Fig. 1).

To start with, we assume that the source σ is strictly monochromatic with wavelength $\lambda = c/\nu$ and generates the illuminating field at the aperture plane $(u, v) = \vec{u}$ with a complex amplitude $E(\vec{u})$. The complex field amplitude at the point of observation $(x, y, z) = (\vec{x}, z)$ is related to the aperture field

$$W(\vec{u}) = E(\vec{u}) \cdot T(\vec{u}, \nu) \tag{1}$$

by the Kirchoff integral

$$w(\vec{x}, z) = \frac{1}{i\lambda} \int_{\Lambda} W(\vec{u}) \frac{\exp(ikL)}{L} \frac{z}{L} \, d\vec{u}, \tag{2}$$

where

$$L = \sqrt{(\vec{x} - \vec{u})^2 + z^2}, \qquad k = \frac{2\pi}{\lambda}. \tag{3}$$

We represent the antenna radiation originating from an imaginary point C (see Fig. 1) by a so-called phase centre:

$$w(\vec{x}, z) = E_0 \frac{\exp(ikR)}{R} D(\vec{\theta}), \quad E_0 = \text{const}. \tag{4}$$

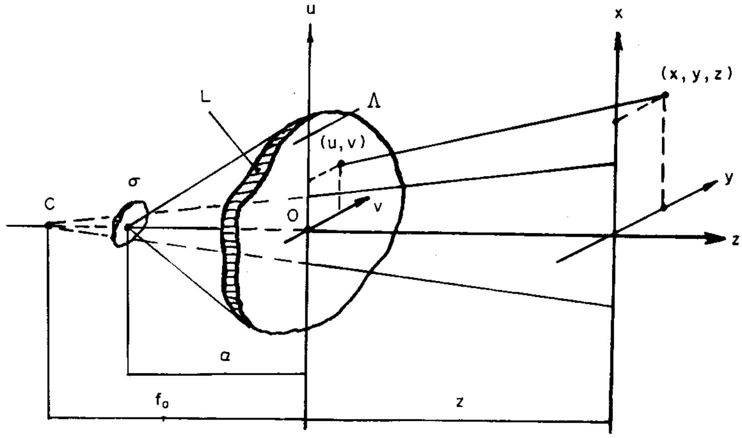

Fig. 1. Geometry of an optical antenna.

where

R is the distnce from the phase centre to the point of observation, and
$\vec{\theta}$ is the angular coordinate of the point of observation reckoned from the phase centre of the antenna.

If f_0 be the distance from the phase centre to the aperture plane, then in (4) we have:

$$\vec{\theta}=\frac{\vec{x}}{R_z}, \qquad R=\sqrt{\vec{x}^2+R_z^2}, \quad R_z=f_0+z. \tag{5}$$

We introduce the corresponding "reduced field" at the aperture, $W_R(\vec{u})$, via

$$W(\vec{u})=E_0\,\frac{\exp(ikf)}{f}\,W_R(\vec{u}), \quad f=\sqrt{\vec{u}^2+f_0^2}. \tag{6}$$

To simplify the work we adopt the paraxial approximation

$$\frac{|\vec{u}|}{z}\ll 1, \qquad \vec{u}\in\Lambda, \qquad \frac{|\vec{x}|}{z}\ll 1. \tag{7}$$

Substituting Eqs (4) and (6) into Eq. (2), we obtain

$$D(\vec{\theta})=\frac{1}{i\lambda f_R}\int_\Lambda W_R(\vec{u})\exp\left[\frac{ik}{2f_R}(\vec{u}-f_0\cdot\vec{\theta})^2\right]\mathrm{d}\vec{u}, \tag{8}$$

where f_R is the reduced distance,

$$\frac{1}{f_R}=\frac{1}{f_0}+\frac{1}{z}. \tag{9}$$

Equations (4) and (6) allow one to "isolate" from the observed field w the point source situated at the phase centre C. The function $D(\vec{\theta})$ defined in Eq. (8) determines the angular resolution of the complex radiation amplitude of the point source. We shall refer to $D(\vec{\theta})$ as the directivity diagram of the optical antenna. Equation (8) shows that the directivity diagram is related to the reduced aperture field by a Fresnel transformation, rather than a two-dimensional Fourier transformation as for radio antennae [1]. In addition, there is here a weak z-dependence of D, which disappears for $z\gg f_0$.

Let us generalize the foregoing to an extended polychromatic source of illumination σ. As is well known [5], the radiation intensity of an optical antenna for partially coherent light, registered by photodetectors, is expressible in terms of mutual coherence functions. Let us introduce the mutual coherence functions of the antenna field $\Gamma_H(\vec{x}_1, z_1, \vec{x}_2, z_2, \tau)$, the illuminating field $\Gamma_0(\vec{u}_1, \vec{u}_2, \tau)$, and their respective spectral densities $G_H(\vec{x}_1, z_1, \vec{x}_2, z_2, \nu)$ and $G_0(\vec{u}_1, \vec{u}_2, \nu)$.

By analogy with (4) we separate the dependence on the angle θ from that on the distance of the phase centre to the point of observation, R:

$$\Gamma_H(\vec{x}_1, z_1, \vec{x}_2, z_2, \tau) = |E_0|^2 \frac{\exp[ik(R_1 - R_2)]}{R_1 R_2} Q_H(\vec{\theta}_1, \vec{\theta}_2, \tau) \tag{10}$$

$$\vec{\theta}_j = \frac{\vec{x}_j}{f_0 + z_j}, \qquad R = \sqrt{\vec{x}_j^2 + (f_0 + z_j)^2}, \qquad j = \overline{1, 2}, \qquad k = \frac{2\pi\nu}{c}. \tag{11}$$

In particular,

$$I(\vec{x}, z) = |E_0|^2 \frac{1}{R^2} Q(\vec{\theta}). \tag{12}$$

We shall refer to the function

$$Q(\vec{\theta}) = Q_H(\vec{\theta}, \vec{\theta}, 0) \tag{13}$$

as the partially-coherent directivity diagram (of the intensity). It determines the optical antenna's radiation intensity distribution for partially coherent light.

We also introduce the "reduced" mutual spectral density $G_R(\vec{u}_1, \vec{u}_2, \nu)$ analogously to (6):

$$T(\vec{u}_1, \nu) \cdot G_0(\vec{u}_1, \vec{u}_2, \nu) \cdot T^*(\vec{u}_2, \nu) = |E_0|^2 \frac{\exp[ik(f_1 - f_2)]}{f_1 f_2} \cdot G_R(\vec{u}_1, \vec{u}_2, \nu), \tag{14}$$

$$f_j = \sqrt{\vec{u}_j^2 + f_0^2}, \qquad j = \overline{1, 2}, \tag{15}$$

where * denotes complex conjugation.

By using the propagation formula for the mutual coherence function [5], one can easily obtain the partially-coherent directivity diagram in the paraxial approximation:

$$Q(\vec{\theta}) = \frac{4}{c^2 f_R^2} \int_0^\infty \nu^2 \, d\nu \int_\Lambda \int_\Lambda G_R(\vec{u}_1, \vec{u}_2, \nu) \cdot \exp\left\{\frac{ik}{2f_R} [(\vec{u}_1 - f_0(\vec{\theta})^2 - (\vec{u}_2 - f_2(\vec{\theta})^2]\right\} d\vec{u}_1, d\vec{u}_2. \tag{16}$$

In particular, for quasimonochromatic light of mean frequency $\bar{\nu}$ one may consider the propagation of the direct mutual coherence $\Gamma_R(\vec{u}_1, \vec{u}_2, 0)$. It can be shown that as one approaches perfect coherence, Eq. (14) reduces to (6), while (16) becomes

$$Q(\vec{\theta}) = |D(\vec{\theta})|^2, \tag{17}$$

where $D(\vec{\theta})$ is the complex amplitude directivity diagram in Eq. (8).

It is particularly interesting to consider optical antenna arrays prepared from synthesized optical elements [4]. For these we have

$$T(\vec{u}, \nu) = \sum_{(n,m) \in I_N} T_{nm} \chi_{nm}(\vec{u}); \quad \vec{u} \in \Lambda, \tag{18}$$

where T_{nm} is the complex transmission coefficient at the centre ξ_{nm} of element Λ_{nm} of the array, and $\chi_{nm}(\vec{u})$ is the complex transmission function of Λ_{nm}. Normally

$$\chi_{nm}(\vec{u}) = \chi^{(0)}(\vec{u} - \vec{\xi}_{nm}), \qquad \chi^{(0)}(\vec{u}) = \begin{cases} 1, & \vec{u} \in \Lambda^{(0)} \\ 0, & \vec{u} \bar{\in} \Lambda^{(0)} \end{cases}.$$

We sum over all pairs I_N of indices (n, m) for which $\Lambda_{nm} \subset \Lambda$ (Fig. 2).

The directivity diagram of an optical antenna array is calculated via Eq. (8) or (16), in which $T(\vec{u}, \nu)$ is determined by (18). As an example, let us quote the result for the partially coherent array diagram when the source is quasimonochromatic:

$$Q(\vec{\theta}) = \frac{4\bar{\nu}^2}{c^2 f_R} \sum_{(n_1, m_1) \in I_N} \sum_{(n_2, m_2) \in I_N} T_{n_1 m_1} \cdot T^*_{n_2 m_2} \cdot \exp\left\{\frac{ik}{2z} (\vec{\xi}^2_{n_1 m_1} - \vec{\xi}^2_{n_2 m_2}) - \frac{ikf_0}{d_R} (\vec{\xi}_{n_1 m_1} - \vec{\xi}_{n_2 m_2})\vec{\theta}\right\} \cdot Y_{n_1 m_1 n_2 m_2}(\vec{\theta}), \tag{19}$$

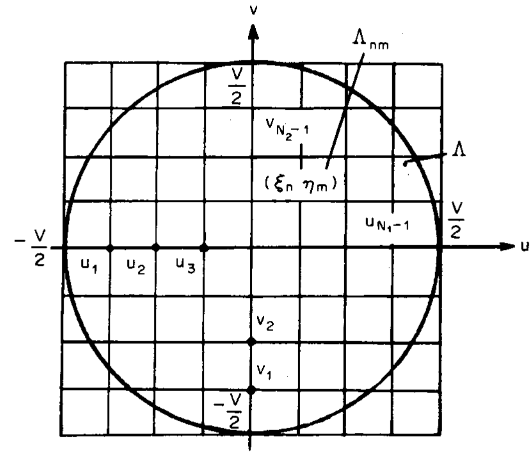

Fig. 2. Geometry of an optical antenna array.

where

$$k=\frac{2\pi v}{c},$$

$$Y_{n_1m_1n_2m_2}(\vec{\theta})=\frac{f_0^2}{|E_0|^2}\int_{\Lambda^{(0)}}\int_{\Lambda^{(0)}}\Gamma_0(\vec{\xi}_{n_1m_1}+\vec{\varepsilon}_1,\vec{\xi}_{n_2m_2}+\vec{\varepsilon}_2,0)$$
$$\times\exp\left\{ik\left[\left(\frac{\vec{\xi}_{n_1m_1}}{z}\right)-\frac{f_0\vec{\theta}}{f_R}\right)\vec{\varepsilon}_1-\left(\frac{\vec{\xi}_{n_2m_2}}{z}-\frac{f_0\vec{\theta}}{f_R}\right)\vec{\varepsilon}_2\right]\right\}d\vec{\varepsilon}_1\,d\vec{\varepsilon}_2. \quad (20)$$

The design problem of an optical antenna is formulated as follows. Given the mutual coherence function or spectral density of the source radiation field, find the directivity diagram for the intensity, $Q_0(\vec{\theta})$. The optical element characteristic at the aperture $T(\vec{u}, v)$ must be so chosen that the approximation

$$Q(\vec{\theta})\cong Q_0(\vec{\theta}) \quad (21)$$

holds.

Hence, the design algorithm of an optical antenna consists of calculating the mutual coherence function Γ_0, and solving the approximation problem (21), where $Q(\vec{\theta})$ is given by one of the relations (8), (16) or (19).

The approximation problem may be solved by well-developed numerical procedures involving Chebyshev approximations to complex functions. However, it must be borne in mind that since the wavelength is small, optical antenna arrays will have a large number of elements, of order $\sim 10^6$–10^8. The design process must involve the use of computers.

Finally, it is important to note that, by using the worked-out design methods and algorithms, one designs on the computer an amplitude mask of a flat optical element which forms several lobes at one of the angular coordinates.

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