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KINOFORM OPTICAL ELEMENTS IN OPTICAL SYSTEM DESIGN OVER A WIDE SPECTRAL RANGE

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Abstract—The paper uses KE as an acronym for kinoform optical element, and considers the design of KE-based apochromatic high-resolution systems for use over a wide spectral range. The aberrational and dispersional properties of KEs and ordinary optical elements are compared. It is demonstrated that combining two regular grades of glass with a KE leads not only to apochromatic correction but can also correct for monochromatic aberrations and the secondary spectrum, as well as for spherochromatism, provided the isoplanatic condition is obeyed. Design methodology and philosophy are discussed for KE-based apochromat–anastigmats that have 30–50% smaller masses than similar systems built around more usual optical elements.

A challenging problem of contemporary optics is to design optical systems capable of high quality image formation at the diffraction limit over a wide spectral range.

It has been shown [1] that kinoform-type synthesized hologram optical elements (inscribed on ordinary glass substrates) [2] can operate effectively over a fairly wide spectral region. It is therefore of great interest to analyse the corrective measures afforded through the use of kinoform optical elements (KE) in the design of fast, high quality optical systems operating with nonmonochromatic radiation. The use of holographic optical elements as corrective devices for monochromatic aberrations has been considered in [3, 4]. The possible use of zone plates (a special case of KE) as correctors of secondary spectra was advocated by Slyusarev [5].

In the present work we consider the prospects of applying KE to the design of apochromatic anastigmat systems employing regular manufactured glass, and to correct residual aberrations arising particularly in faults of the manufacturing process.

In order to exhibit the dispersion properties of KE and its advantageous position on the $(p\text{–}\nu)$-diagram, which relates the Abbe number and the relative dispersion ratio $p = \nu'/\nu$, we shall consider the generalized Abbe invariant [6]. The generalized Abbe invariant for an axisymmetric surface with a KE has the form

$$Q = n\left(1 - c\left(1 - m\mu\frac{n_0}{n}\right) - m\mu\frac{n_0}{n}\, l_0\right), \tag{1}$$

where c is the curvature of the KE surface, $l = 1/S$, $l_0 = 1/S_0$, S and S_0 are the read/write source positions for the holographic optical element, n and n_0 are the refractive indices at wavelengths λ and λ_0 respectively, and m is the order of diffraction.

Hence the power of the surface with the KE is expressible as

$$\varphi = C(n' - n - m\mu(n'_0 - n_0)) + m\mu\varphi_0, \tag{2}$$

where $\varphi_0 = n'_0 l'_0 - n_0 l_0$.

For a KE operating in air, whether on transmission or reflection, the expression for the power may be simplified as follows:

In the transmission mode $(n = n' = n_0 = n'_0 = 1)$ one has

$$\varphi = \mu\varphi_0, \qquad m = 1. \tag{3}$$

In the reflection mode $(n = -n' = n_0 = -n'_0 = 1)$ we get

$$\varphi = -2C(1 - \mu) + \mu\varphi_0, \qquad m = 1. \tag{4}$$

The first chromatic sum for a thin lens has the well-known form

$$S_{\mathrm{I}}^{\mathrm{chr}} = h^2\delta\varphi = h^2\frac{\varphi}{\nu}$$

where h is the height of the first paraxial ray, φ is the (optical) power, and ν is the Abbe number.

An analogous expression holds for a KE, in keeping with (3) and (4):

$$S_{\mathrm{I}}^{\mathrm{chr}} = h^2\delta\mu\varphi \tag{5}$$

and

$$S_{\mathrm{I}}^{\mathrm{chr}} = h^2\delta\mu\varphi(1 + 2C/\varphi). \tag{6}$$

So for a KE the number ν is determined as follows:

For transmission,

$$\nu = \frac{\lambda_0}{\delta\lambda} \tag{7}$$

and in the reflection mode

$$\nu = \frac{\lambda_0}{\delta\lambda}\frac{1}{(1 + 2C/\varphi)}. \tag{8}$$

In the reflection mode, if $C = -\varphi/2$, a KE has no dispersion, while if $C = 0$ its dispersion properties are the same as in the transmission mode. We remark that if $C \neq 0$ and $\varphi_0 = 0$, such a KE is also dispersive, but in this event ν is not determined and

$$S_{\mathrm{I}}^{\mathrm{chr}} = h^2 2C\delta\mu. \tag{9}$$

In order to present clearly the prospects for getting qualitatively new results by the combination of a KE with regular glasses, enabling one to produce apochromatic systems, let us consider the position of the KE on the (p–ν) diagram.

The relative dispersion ratio for the KE is

$$p = \nu'/\nu = \delta\lambda/\delta\lambda'. \tag{10}$$

For example, $\nu_D = -3.46$; $p_{F,D} = 0.61$ and $\nu_{D,C} = 0.39$. A schematic (p–ν) diagram is shown in Fig. 1. It shows that the KE occupies quite a special position and is considerably further from the "normal curve" than a fluorite type crystal. In addition, there is no single optical glass or crystal which has this relative dispersion ratio. One therefore appreciates that it is impossible to construct an apochromatic combination of a KE with a single grade glass. It is much more promising to combine a KE with two grades of glass, where we emphasize that regular glasses are involved.

To calculate the optical power of apochromates one normally tries to solve a system of three equations, which provides for the superposition of three colour images, but then the longitudinal chromatic aberration at intermediate wavelengths cannot be controlled and may attain undesirably large values. The simultaneous satisfaction of four equations partially overcomes the difficulties and leads to a so-called four-colour apochromate [7]. The most rational option for the optical power is via the least squares method, which makes allowance for the defocusing and spectral coefficients of the actinic flux forming the image.

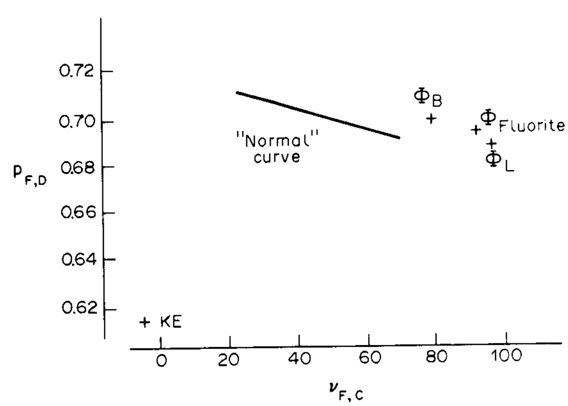

Fig. 1.

The system of equations for determining the power assumes the following form:

$$\sum_{i=1}^{n} \sum_{\lambda} (T_\lambda a_{i\lambda} a_{j\lambda}) \varphi_i = \sum_{\lambda} a_{j\lambda} T_\lambda, \tag{11}$$

where

φ_i is the power of the elements at the primary wavelength,
$a_{i\lambda} = (n_{i\lambda} - 1)/(n_i\lambda_0 - 1)$ for the lens,
$a_{i\lambda} = \lambda/\lambda_0$ for the KE.

The second problem we have to consider in designing an objective with a KE has to do with rectifying spherochromatic aberration, since an uncorrected spherochromatic aberration may lead to none of the results obtained by apochromatic correction of the secondary spectra. We must note that the spherical aberration of an object with a KE may always be eliminated for a single (say, the primary) colour, so that

$$S_{0\mathrm{I}}^{\mathrm{HOE}} + \sum_{i=1}^{2} S_{0\mathrm{I}}^{(i)} = 0, \tag{12}$$

where $S_{\mathrm{I}}^{(i)}$ is the coefficient of spherical aberration of the ith lens component.

Since the power of the KE employed to obtain the apochromatic correction is far smaller than the power of the lens, the dispersion-related change of the coefficient of spherical aberration of the KE [8] is insignificant and may be neglected. Thus the coefficient of third-order spherical aberration for an objective at arbitrary wavelengths will be written as

$$S_{\mathrm{I}} = S_{\mathrm{I}}^{(1)} + S_{\mathrm{I}}^{(2)} - \mu(S_{0\mathrm{I}}^{(1)} + S_{0\mathrm{I}}^{(2)}), \tag{13}$$

which, when differentiated, gives

$$\mathrm{d}S_{\mathrm{I}} = \sum_{i=1}^{2} (\mathrm{d}S_{\mathrm{I}}^{(i)} - \mathrm{d}\mu S_{0\mathrm{I}}^{(i)}). \tag{14}$$

The coefficient of spherical aberration for a thin lens has been obtained on the basis of the Czapski–Eppenstein formula in [9]. We shall employ a more symmetrical expression in terms of Coddington variables [10]. After fairly cumbersome transformations we obtain for (14):

$$\mathrm{d}S_{\mathrm{I}} = h^4 \sum_{i=1}^{2} \varphi_i^3 \{A_i X_i^2 + B_i X_i Y_i + C_i Y_i^2 + D_i X_i + E_i Y_i + G_i\}, \tag{15}$$

where

$$A = \frac{1}{v}\frac{n^2+2}{4n^2(n-1)^2} - \frac{1}{v_{\mathrm{H}}}\frac{n+2}{4n(n-1)^2};$$

$$B = -\frac{1}{v}\frac{n^2+1}{n^2(n-1)} + \frac{1}{v_{\mathrm{H}}}\frac{n+1}{n(n-1)};$$

$$C = \frac{1}{v}\frac{3n^2+2}{4n^2} - \frac{1}{v_{\mathrm{H}}}\frac{3n+2}{4n};$$

$$D = -L\frac{2(n+1)}{n(n-1)} - \frac{1}{v}\frac{n+1}{n(n-1)};$$

$$E = L\frac{3n+2}{n} + \frac{1}{v}\frac{3n+2}{2n};$$

$$G = \frac{1}{v}\frac{n(3n-2)}{4(n-1)^2} - \frac{1}{v_{\mathrm{H}}}\frac{n^2}{4(n-1)^2};$$

$L = \mathrm{d}l/\varphi$;

X_1, X_2 are lens shape parameters (of sag);

$X = (C_1 + C_2)/(C_1 - C_2)$;

v_1, v_2, and v_{H} are Abbe numbers for the lens and KE, respectively;

Y_1, Y_2 are conjugation parameters, $Y = (l' + l)/(l' - l)$.

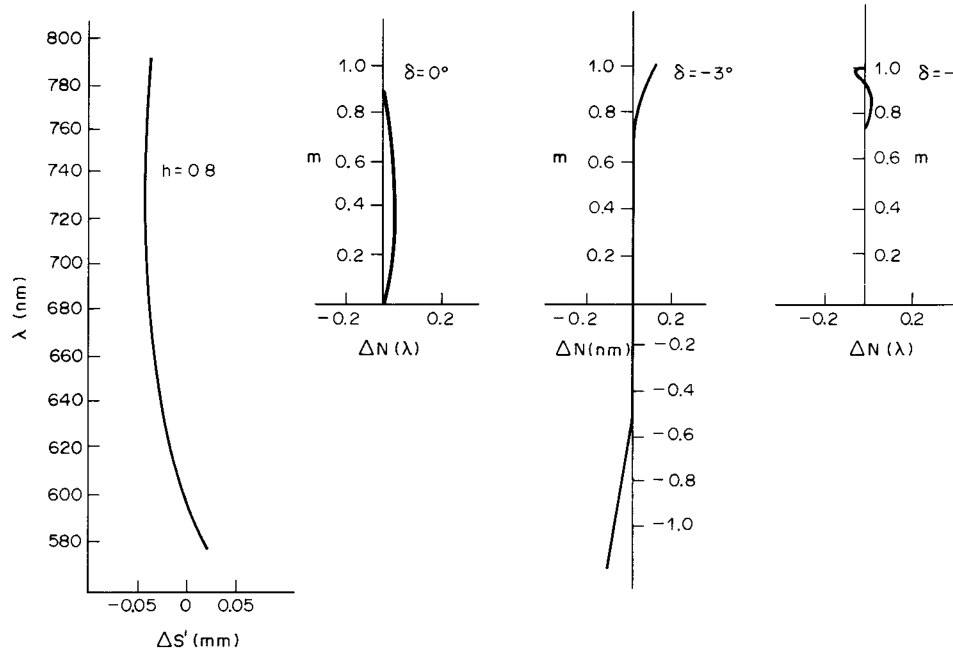

Fig. 2.

For given power and object distance $dS_{\rm I}$ in (15) is a quadratic form in X_1 and X_2. The spherical aberration may therefore be eliminated for a whole family of X_1, X_2 by satisfying the condition

$$dS_{\rm I} = 0. \tag{16}$$

Numerical studies have shown that the solutions sets of (16) are two branches of a hyperbola. The condition for eliminating third order coma,

$$\sum_{i=1}^{3} S_{\rm II}^{i} = 0 \tag{17}$$

gives, for a thin component with a KE, a straight line solution in the X_1, X_2 plane as the general solution of (16) and (17). For example, for K8, Φ1 glasses: $X_1 = -0.14$, $X_2 = 1.676$ and $X_1 = 1.75$, $X_2 = -7.18$ respectively. Therefore, the simultaneous elimination of secondary spectra and spherochromatic aberration of a two-lens objective with KE may be combined with the satisfaction of the isoplanatism condition.

It is well known that astigmatism and field curvature of the image in a thin system cannot be completely eliminated. But by employing a plano-astigmatic compensator, such as a meniscus [11] or a three-lens system [12], one can produce an apochromat–anastigmat. These methods are the basis of a whole family of apochromat-teleobjectives which do not contain special glass or crystals, and which are 30–50% lighter than comparable objectives utilizing special glasses or crystals. Secondary spectra and aberration of a broad beam falling on one such objective are shown in Fig. 2. The parameters are: $f' = 1000$, $2\beta = 6°$, relative aperture 1:8.

The lens plus KE component we have discussed can be utilized to construct various kinds of apochromat–anastigmats to serve as front components of an objective with variable focal distance, and similar systems.

Finally, we consider ways to correct remanent monochromatic aberrations due to manufacturing faults, in particular, nonuniformity in glass, and errors in the optical surfaces employed in constructing the holographic corrector. The required topography of the wavefront deformation may be obtained by reconstructing it from the computer analysis of the interferogram [13].

The optical power (or carrier frequency, in hologram terminology) required in the hologram–compensator construction can be selected by satisfying the condition for either apochromatic or zero-correction (Fourier hologram). The resulting maximal spatial frequency on the hologram surface will be too small to allow the construction of phase profiles of kinoform-type holographic optical elements without substantial light diffusion. Such a hologram–compensator has very high diffraction efficiency, does not require the introduction of additional elements into the optical system being corrected, and its position in the optical system can be effectively utilized to balance the aberration over the field of vision.

We conclude that the new class of apochromats based on kinoform optical elements enables one to design high-resolution optical systems capable of producing images within reach of the diffraction limit, over a wide spectral region, which will find application in solving a series of problems in pure and applied optics.

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