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CORRECTION OF FOCUSER PHASE FUNCTION BY COMPUTER-EXPERIMENTAL METHODS

N. L. KAZANSKII

Abstract—Within the Fresnel–Kirchhoff approximation, consideration is given to the distribution of light in the focuser's focal plane into a narrow longitudinal interval along the optical axis. To improve the uniformity of intensity distribution along the focusing interval an iteration procedure is suggested for the correction of the focuser phase function. The results of a computer experiment with the original and the corrected optical elements are presented.

In the design of flat optics elements (FOE) [1] one typically calculates phase characteristics by geometrical methods, whereas the light fields formed by the FOE can only be investigated in terms of a diffraction model. In order to evaluate the FOE performance in the design stage one must investigate and compensate for diffraction effects associated with the design and production of FOE technology.

In the present work a focuser phase function correction is considered for a focuser with an enhanced depth of focus, in order to compensate for diffraction effects generated by radial phase function discretization.

FOCUSER WITH ENHANCED DEPTH OF FOCUS

Let a light beam of complex amplitude $E(r)$ be incident on a radially symmetric optical system (Fig. 1) possessing a phase transmission function $T(r)$. We shall assume that the optical system consists of a classical continuous converging lens of focal length f_0, and that a nearby focuser concentrates radiation in a narrow longitudinal cylinder along the optic axis. The transmission phase function is determined by the set of differential equations [2]:

$$\begin{gathered} b'(r) + \frac{\lambda}{2\pi}\cdot\varphi'(r) = -\frac{r}{\sqrt{r^2 + (f_0 + z(r))^2}} \\ w(z)z'(r) = B_0^2(r)\cdot 2\pi r \\ z(0) = 0, \qquad z(a) = \kappa. \end{gathered} \tag{1}$$

Here, $\{r, \alpha\}$ are polar coordinates in the focuser plane with the origin at its centre; $w(z)$ is the linear power density in the focusing interval $[0, \kappa]$; $B_0(r)$ and $b(r)$ are, respectively, the amplitude and phase of the light beam incident on the focuser; λ is the wavelength of light and a is the focuser radius. For $f_0 \gg a$, a plane illuminating beam $E(r)$, and a constant density $w(z) = \text{const}$, we obtain [2]

$$\varphi(r) = \varphi_0 - \frac{k}{2c}\ln[2c\sqrt{r^2 + (f_0 - cr^2)^2} + 2c^2r^2 + 1 - 2f_0c], \tag{2}$$

where

$$c = \kappa/a^2, \qquad k = 2\pi/\lambda.$$

COMPUTER EXPERIMENT

In order to investigate the operation of focusing optical elements one must calculate the light fields produced by them in the focal region. In the process one must bear in mind the finite resolution δ over the FOE plane, and the finite number of quantum phase levels. In the manufacture of radially symmetric FOE by means of circularly scanning plotters, the quantity δ is determined by the minimum width of the annular photo-mould which is the "spot size" of the photo plotter. We

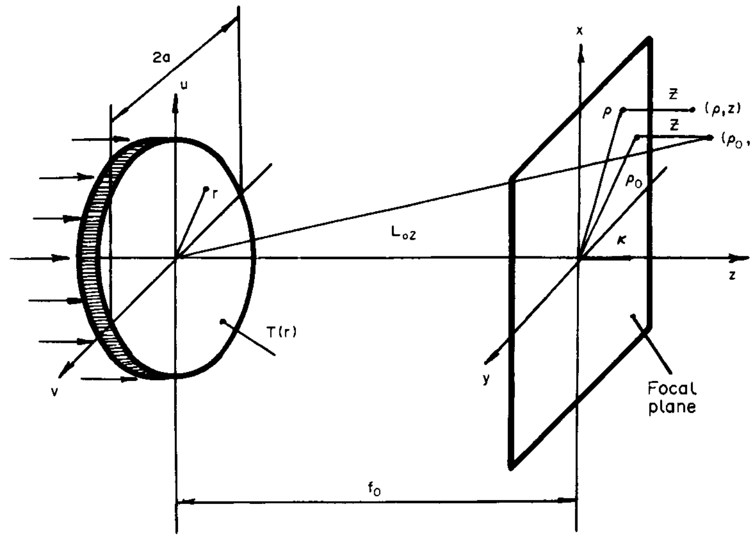

Fig. 1. Optics of FOE operation.

therefore employ the following model phase function in the design of the radially symmetric FOE:

$$\hat{\varphi}(r)=\sum_{p=1}^{N}\varphi_p\cdot\operatorname{rect}\left(\frac{r-r_{p-1/2}}{\delta}\right), \tag{3}$$

where

$N=[a/\delta]$ is the total number of resolved rings on the optical element,
φ is the phase of the pth ring ($p=1\to \mathrm{N}$), and
$r_{p-1/2}=(p-1/2)\delta$ is the radius of the centre of the pth ring.

Remembering that $\delta\gg\lambda$, we use the Kirchhoff integral to estimate the complex amplitude $w(\rho,z)$ at the point of observation (ρ,z), where $\rho=\sqrt{x^2+y^2}$ (*cf* Fig. 1). If $2a\ll f_0$, it can be shown [3] that the complex field amplitude $w(\rho,z)$ near the basal point (ρ_0,z) may be represented in terms of a superposition of fields formed by the individual annuli (r_{p-1},r_p), $p=1\to\mathrm{N}$:

$$\hat{w}(\rho,z)=\frac{k}{\mathrm{i}L_{0z}}\cdot\exp\left[\mathrm{i}k\left(L_{0z}+\frac{\rho^2-\rho_0^2}{2L_{0z}}\right)\right]\cdot\sum_{p=1}^{N}E(r_{p-1/2})\cdot\exp\left(\mathrm{i}\hat{\varphi}_p\right)\int_{r_{p-1}}^{r_p}\exp\left(-\frac{\mathrm{i}qr^2}{2a^2}\right)\cdot J_0\left(\frac{sr}{a}\right)r\,\mathrm{d}r, \tag{4}$$

where

$$r_p=p\cdot\delta,\qquad p=\overline{1,N};\qquad L_{0z}=\sqrt{(f_0+z)^2+\rho_0^2};$$

$$q=-ka^2\left(\frac{1}{L_{0z}}-\frac{1}{L_0}\right);\qquad s=\frac{ka\rho}{L_{0z}};\qquad L_0=\sqrt{\rho_0^2+f_0^2}.$$

One way of doing the integral in (4) is by using the Lommel function [4], whereby the field of each annulus $r\in[r_{p-1},r_p]$ is expressed as the difference of fields from two apertures of radii r_p and r_{p-1}, namely,

$$\hat{w}(\rho,z)\sum_{p=1}^{N-1}[E(r_{p-1/2})\exp(\mathrm{i}\hat{\varphi}_p)-E(r_{p+1/2})\exp(\mathrm{i}\hat{\varphi}_{p+1})]\cdot w_p(\rho,z)+E(r_{N-1/2})\cdot\exp(\mathrm{i}\hat{\varphi}_N)\cdot w_N(\rho,z), \tag{5}$$

where $w_p(\rho,z)$ is the complex amplitude of the light wave incident at the point of observation on the aperture with radius r_p, $p = 1 \to N$. As shown in [4], $w_p(\rho, z)$ is given by the following expression

$$w_p(\rho, z) = \frac{kr_p^2}{iq_p} \exp\left[ik\left(L_{0z} + \frac{\rho^2 - \rho_0^2}{2L_{0z}} \right) \right] \times \begin{cases} \exp\left(\frac{iq_p}{2}\right) \cdot U_1(q_p, s_p) + i \exp\left(-\frac{iq_p}{2}\right) U_2(q_p, s_p) & \text{with } \left|\frac{q_p}{s_p}\right| \leqslant 1, \\ -i \exp\left(\frac{s_p^2}{2q_p}\right) + \exp\left(\frac{iq_p}{2}\right) V_0(q_p, s_p) - \exp\left(-\frac{iq_p}{2}\right) V_1(q_p, s_p) & \text{with } \left|\frac{q_p}{s_p}\right| > 1, \end{cases} \tag{6}$$

where

$$q_p = -kr_p^2\left(\frac{1}{L_{0z}} - \frac{1}{L_0}\right), \qquad s_p = \frac{kr_p\rho}{L_{0z}},$$

and U_n and V_n are Lommel functions [4].

Results of computer experiment

The algorithm for calculating the diffraction integral is implemented via a complex computer programme combined with a software package of image processing for digital holography [5] and complex graphics, called GRAFOR. The focuser computer experiment within a narrow longitudinal cylinder along the optic axis, carried out with the aid of the above package, has yielded the following characteristic features for the light distribution:

(a) an elongated focal region with depth $\sim(0.7\text{–}0.9)\kappa$;
(b) an intensity drop-off towards the annulus of the "focusing interval";
(c) a considerable drop in intensity along the interval $[0, \kappa]$, related to diffraction effects on the focuser ring structure.

Figure 2 illustrates isophotes of the light distribution in the focal region of the focuser with an extended depth of focus, using the parameters: $f_0 = 300$ mm, $\lambda = 0.633$ μm, $a = 12.8$ mm, $\kappa = 15$ mm, $N = 128$. Curve b of Fig. 3 is a graph of the intensity distribution along the optic axis with the same set of parameters. The normalization was fixed at the intensity of the objective's focus. The mean square intensity deviation along the "focusing interval" was 0.374, and the maximal relative deviation $\Delta I/\bar{I} = 0.9$, where $\bar{I}$ is the mean intensity along the interval.

CORRECTIVE PROCEDURES

The computer experimental results require corrections to the focuser phase function based on the generated light intensity distributions. This is effected by means of the design algorithm and software for the optical element which concentrates incident radiant energy along the optical axis into a narrow cylinder on it. The design algorithm is built on the numerical solution of the system of equations (1) with an arbitrary $w(z)$. The iteration procedure for the focuser phase function correction consists of generating a new function $w(z)$ on the basis of the computer experimental data of the previous stage. Thus

$$w_{k+1}(z) = w_k(z) + h \cdot (\bar{I}_k - I_k(z, 0)), \quad z \in [0, \kappa]. \tag{7}$$

Here $w_k(z)$, $w_{k+1}(z)$ are the kth and $(k+1)$th iterations of the linear power density; $I_k(z, 0)$ and $\bar{I}_k$ are the kth iterations of the intensity distribution and mean intensity along the interval $[0, \kappa]$; h is an optimizing parameter chosen so as to minimize the mean square deviation of the intensity along the interval,

$$\varepsilon_{k+1} = \sqrt{\frac{\int_0^\kappa (\bar{I}_{k+1} - I_{k+1}(z, 0))^2\, dz}{\kappa \bar{I}_{k+1}^2}}. \tag{8}$$

The computer experiment is interactive, and may be interrupted as soon as satisfactory results appear on the display screen. The curve in Fig. 3 represents the intensity distribution obtained

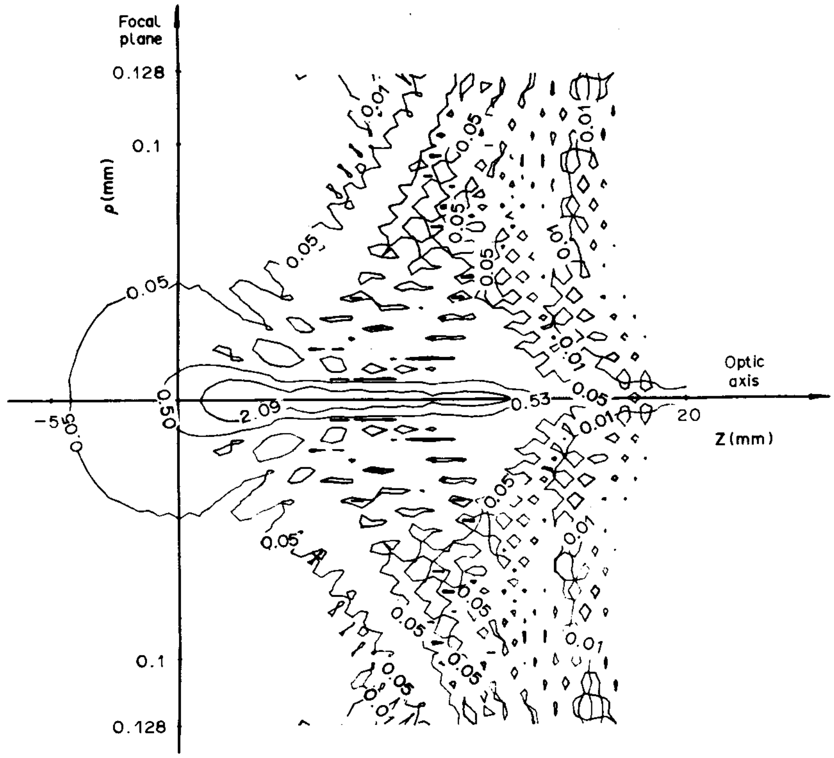

Fig. 2. Isophotes of three-dimensional intensity distribution $I(z, \rho)/I(0, 0)$ in the focal region of an extended-λ focal-depth focuser with $\kappa = 15$ mm, and using $f_0 = 300$ mm, $a = 12.8$ mm, $\lambda = 0.633$ μm, $N = 128$.

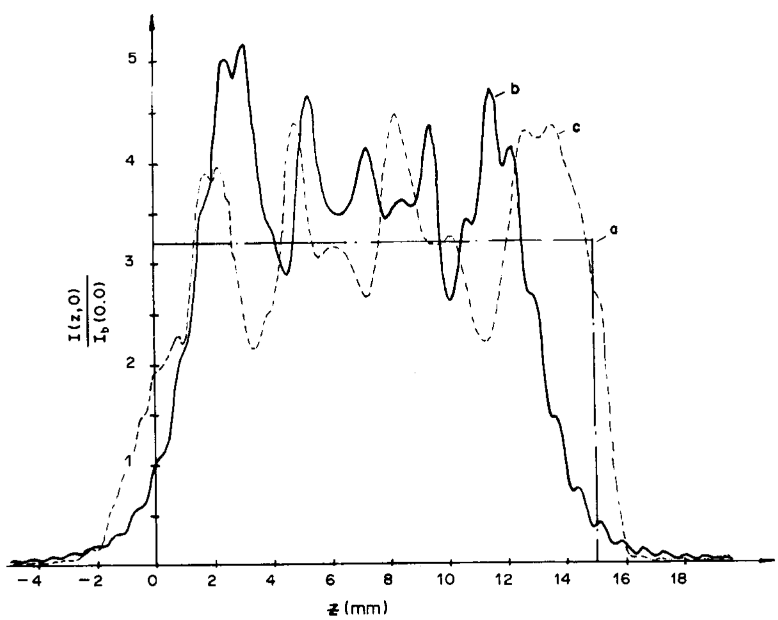

Fig. 3. Intensity distribution $I(z, 0)/I_b(0, 0)$ along the optic axis: (a) "ideal focuser", (b) the focuser in (2), (c) focuser with corrected phase function.

after three iterations. If the intensity level is lowered from 3.21 to 3.05, i.e. by less than 5%, the mean square deviation of the intensity becomes 0.26, and the maximum deviation from the mean $\Delta I/\bar{I} = 0.53$.

CONCLUSION

The computer experiment, based on diffraction relations and operated with a software package, allows for visualization and quantitative analysis of complex light distributions in the focal plane of optical elements, as well as performing corrective procedures for optimizing the phase function of extended-focal-depth focusers. The results enable one to design the corresponding FOE and to study their performance in optical systems.

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