Computer Optics Vol. 1, No. 1, pp. 79–83, 1989
Printed in Great Britain

0955–355X/89 $10.00 + 0.00

WAVEFRONT FORMATION IN GRADIENT WAVEGUIDES WITH SMALL RADIUS BENDS

G. L. ESAYAN and S. G. KRIVOSHLYKOV

Abstract—Propagation of light radiation through sharply bent units of multimode waveguides with quadratic refractive index profiles is theoretically investigated. Expressions have been derived for coefficients of mode coupling and for the energy loss on the bends. Using the results obtained it is proposed to use bent waveguides as mode filters operating in the matched mode and also as adjustable-sensitivity refractometers.

With the development of optical fibre communications and integrated optics increasing attention has been given to light propagation in bent guides.

Practical guides contain bends which cause a redistribution of energy among the modes. This in turn influences the intermodal dispersion, and leads to signal attenuation and energy transfer into incoherent radiation, resulting in losses. Bent guides have been studied in the past in order to investigate bend-related losses [1–4]. An analysis of the influence of small deviations of the axis of a graded fibre axis from a straight line on the coupling of its modes has been given in [5].

The aim of the present work is to study mode coupling in a multimode guide with a parabolic index profile whose axis is bent into a curve of radius r.

EFFECT OF REFRACTIVE INDEX PROFILE

Consider a plane optical guide with a parabolic refractive index profile

$$\begin{aligned} n^2(x) &= n_0^2 - \omega^2 x^2 & |x| \leqslant a \\ n^2(x) &= n_0^2 - \omega^2 a^2 = n_1^2 & |x| > a. \end{aligned} \tag{1}$$

It was shown in [1] that if the guide characterized by (1) is bent round a radius r, then light propagation along it may be discussed in terms of an equivalent rectilinear guide with the effective index profile,

$$n_{\mathrm{eff}}^2(x) = n^2(x)\left(1 + \frac{x}{r}\right)^2, \tag{2}$$

chosen in such a way, that the transverse field distribution in the bent and effective guides are identical. Figure 1 shows both the parabolic and effective index profiles, (1) and (2). The latter may conveniently be represented by

$$\begin{aligned} &n_{\mathrm{eff}}^2(x') = n_0'^2 - \omega'^2 x'^2 - \left(1 + \sqrt{1 + 8\frac{n_0^2}{\omega^2 r^2}}\right)\frac{\omega^2}{r} x'^3 - \frac{\omega^2}{r^2} x'^4, \quad |x| \leqslant a \\ &n_{\mathrm{eff}}^2(x') n_1^2\left(1 + \frac{x' - \Delta_{\mathrm{eff}}}{r}\right), \quad |x| > a, \end{aligned} \tag{3}$$

where

$$x' = x + \Delta_{\mathrm{eff}};$$

$$\Delta_{\mathrm{eff}} = \frac{r}{4}\, 1 + 8\frac{n_0^2}{\omega^2 r^2};$$

$$n_0' = n(\Delta_{\mathrm{eff}})\left(1 + \frac{\Delta_{\mathrm{eff}}}{r}\right);$$

$$\omega' = \left[\omega^2 - \frac{n_0^2}{r^2} - 6\omega^2\left(\frac{3 + \sqrt{1 + 8\frac{n_0^2}{\omega^2 r^2}}}{4}\right) + 6\omega^2\left(\frac{3 + \sqrt{1 + 8\frac{n_0^2}{\omega^2 r^2}}}{4}\right)^2\right]^{1/2} \tag{4}$$

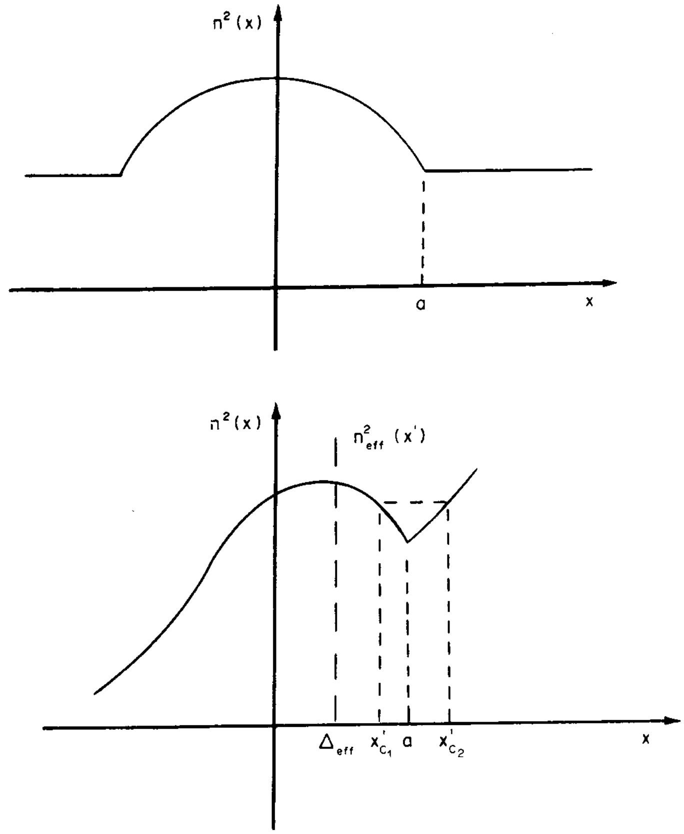

Fig. 1.

This shows that radiation propagating in an axially bent waveguide is shifted towards the guide boundary by an amount which depends on $\Delta_{\text{eff}}(r)$. The depth of the effective index profile decreases with the bend radius, which results in the transformation of higher guided modes into radiation modes. At a certain critical radius r_{cr} all the radiation leaks from the guide.

An estimate for r_{cr} was given in [3]. This clearly corresponds to the case in which $\Delta_{\text{eff}}(r)$ equals the half-width of the guide a. If $|x'| \leqslant a - \Delta_{\text{eff}} = a_{\text{eff}}$, we have

$$\left(1 + \sqrt{1 + 8\,\frac{n_0^2}{\omega^2 r^2}}\right)\frac{\omega^2}{r}\,x'^3 + \frac{\omega^2}{r^2}\,x'^4 \ll 1 \tag{5}$$

and the effective refractive index profile of the equivalent rectilinear light guide may, to the first order, be approximated by the parabolic form

$$n_{\text{eff}}^2(x') = n_0'^2 - \omega'^2 x'^2. \tag{6}$$

Hence the study of light propagation in a bent guide having a parabolic refractive index profile in fact reduces to the analogous problem of off-axial butting of waveguides with various gradient parameters, as discussed in [6, 7].

It is known that, in the paraxial approximation, in a parabolically graded guide, the beam trajectory oscillations are the same for all beams, irrespective of their amplitudes [5, 8]. In other words, the cell in the coordinate-momentum phase space associated with the incoming beam is transported without deformation of its boundaries. At distances which are multiples of the period of oscillation of the beam trajectory the transfer field distribution returns to the critical one. However, relinquishing the paraxial treatment and taking into account deviations from the parabolic profile, the beam trajectory oscillations will become amplitude dependent, resulting in the rephasing of the beam and the averaging of its phase over some distance z_{M}. As a result, the phase cell diffuses into an annulus in phase space, a shape which is preserved in the subsequent propagation of the radiation along the waveguide (Fig. 2). However, in the short initial section $z \ll z_{\text{M}}$, rephasing of beam occurs in a definite cross-section and the field reverts to the incident field.

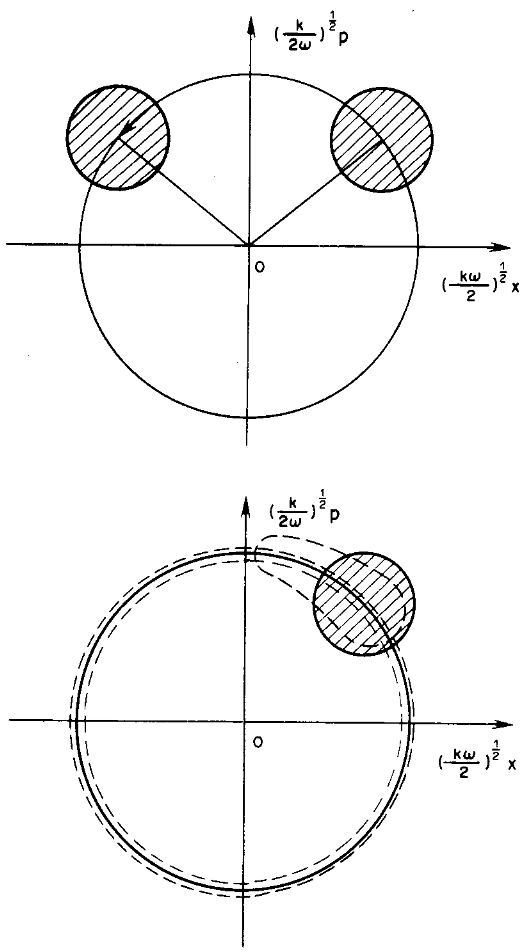

Fig. 2.

This phenomenon may be exploited to construct mode filters operating in the matched mode, wherein the final widths of the mode and the beam are constant, while the final trajectories of the mode and beam centres have the same functional forms as initially. The matching is periodic with a period equal to the beam oscillation period in the bend, whereby the length of the bent section l must satisfy $l \ll z_M$.

At distances $z \gg z_M$ information about the beam phase is lost, and the field is conveniently described in terms of modes whose coupling coefficients are determined by the mode composition of the final radiation.

MODE COUPLING COEFFICIENTS

We use the results of [6, 7] to discuss the mode coupling coefficients. The coupling coefficients W_n^m between modes of the bent section $|n'\rangle$ and the initial modes $|m\rangle$ are determined by the squared moduli of the overlap integrals $T_{n'}^m = \langle n' \mid m \rangle$ whose explicit form is given in [6, 7]. For our problem these are

$$T_{n'}^m = T_0^0 (m!n'!)^{-1/2} H_{mn'}(\sigma, \tau)$$

$$T_0^0 = \left(\frac{2\sqrt{\omega\omega'}}{\omega + \omega'}\right)^{1/2} \exp\left\{-\frac{k}{2}\frac{\omega\omega'}{\omega + \omega'}\Delta_{\text{eff}}^2\right\}, \tag{7}$$

where

$$\sigma = \frac{\omega'}{\omega + \omega'} (2\kappa\omega)^{1/2} \Delta_{\text{eff}}; \qquad \tau = -\frac{\omega}{\omega + \omega'} (2\kappa\omega')^{1/2} \Delta_{\text{eff}} \tag{8}$$

and $H_{mn}(\sigma, \tau)$ is the two-variable Hermite polynomial.

The following recurrence relations between the overlap integrals are useful in calculations:

$$T_{m+1}^{n'} = \frac{\eta}{\xi}\left(\frac{m}{m+1}\right)^{1/2} T_{m-1}^{n'} + \frac{1}{\xi}\left(\frac{n'}{m+1}\right)^{1/2} T_m^{n'-1} + \frac{1}{\xi}\frac{\delta}{\sqrt{m+1}}\frac{\omega'}{\omega} T_m^{n'}, \tag{9a}$$

$$T_m^{n'+1} = -\frac{\eta}{\xi}\left(\frac{n'}{n'+1}\right)^{1/2} T_m^{n'-1} + \frac{1}{\xi}\left(\frac{m}{n'+1}\right)^{1/2} T_{m-1}^{n'} - \frac{1}{\xi}\frac{\delta}{\sqrt{n'+1}} T_m^{n'}, \tag{9b}$$

$$T_{m+1}^{n'} = \frac{\xi}{\eta}\left(\frac{m}{m+1}\right)^{1/2} T_{m-1}^{n'} - \frac{1}{\eta}\left(\frac{n'+1}{m+1}\right)^{1/2} T_m^{n'+1} - \frac{1}{\eta}\frac{\delta}{\sqrt{m+1}}\left(\frac{\omega'}{\omega}\right)^{1/2} T_m^{n'}, \tag{9c}$$

$$T_m^{n'+1} = -\frac{\xi}{\eta}\left(\frac{n'}{n'+1}\right)^{1/2} T_m^{n'-1} + \frac{1}{\eta}\left(\frac{m+1}{n'+1}\right)^{1/2} T_{m+1}^{n'} - \frac{1}{\eta}\frac{\delta}{\sqrt{n'+1}} T_m^{n'}, \tag{9d}$$

where

$$\xi = \frac{\omega + \omega'}{2\sqrt{\omega\omega'}}; \qquad \eta = \frac{\omega - \omega'}{2\sqrt{\omega\omega'}}; \qquad \delta = \left(\frac{k\omega}{2}\right)^{1/2} \Delta_{\text{eff}}. \tag{9e}$$

The coupling coefficients between modes of the initial and final rectilinear sections of the waveguide will be determined over the length of the bent section by the expressions

$$W_n^{m''} = \sum_{l'} W_n^{l'} W_{l'}^{m''}, \tag{10}$$

where $W_n^{l'}$ are the coupling coefficients between modes of the initial rectilinear and the bent sections of the guide. given by (7), and $W_{l'}^{m''}$ are the coupling coefficients between modes in the bent and in the final rectilinear sections, computable via (7) by interchanging ω and ω' and putting

$$\Delta_{\text{eff}} = \left(1 - \sqrt{1 + 8\frac{n_0^2}{\omega^2 r^2} r/4}\right).$$

The summation in (10) is performed over all the guided modes of the bent section.

By looking at the form of the coupling constants it is simple to show that a continuous and fairly slow change (i.e. $d\Delta_{\text{eff}}/dz \ll 1$) of the bending radius does not result in a redistribution of energy among the modes. This phenomenon affords further possibilities for utilizing bent fibres as mode filters operating in the matched mode. Such filters will consist of a fibre bent into a spiral, the number of guided modes being determined by the minimum radius of the curved spiral.

LOSSES

Losses in waveguide bends are caused by a redistribution of the energy among the waveguide modes, as a result of which part of the energy in the guided modes is transferred into radiation modes. If a light beam of energy P_0 is propagated along the initial section of the waveguide, then the energy of the guided radiation arriving into the final straight section may be estimated by the formula

$$P_{\text{oct}} = \sum_{m=0}^{M} \sum_{n''=0}^{N} P_m W_m^{n''}, \tag{11}$$

where M is the number of initially excited modes, N the number of guided modes in the given waveguide, and P_m is the energy of the mth initial mode, so that $\sum_{m=0}^{M} P_m = P_0$.

A quantitative estimate in a real waveguide having a gradient parameter $\sim 7 \times 10^{-3}\ \mu m^{-1}$ and width $2a \sim 70\sigma\ \mu m$ indicates that the redistributed energy, and accordingly the losses, become significant at radii of order $r \sim 2 \times 10^4\ \mu m$. For smaller bend radii losses are increased rapidly, and by the time $r \sim 2 \times 10^2\ \mu m$ all radiation will have escaped the waveguide.

Other losses, due to tunnelling of waves through the external boundary of the light guide, have been considered in [4]. The energy lost by tunnelling in a particular mode depends on the refractive index of the external medium and increases with mode order. This means that bent waveguides can be used as tunable refractometers whose sensitivity can be altered by measuring the energy of the various modes as it leaves the waveguide. In practice the selection of a specified transverse mode from the light beam leaving the waveguide may be carried out by using spatial filters synthesized on the computer [10, 11].

REFERENCES

1. M. Heilbloom and J. H. Harris. *IEEE J. Qe* **11**, 75 (1975).
2. D. Marcuse. *J. Opt. Soc. Am.* **66**, 311 (1976).
3. S. K. Morshnev and A. V. Frantsesson. *Kvantovaya Elektronika* **9**, No. 2 (1982).
4. C. Winkler, J. D. Love and A. K. Ghatak. *Opt. Quantum Elec.* **11**, 173 (1979).
5. Ch. G. Unger. *Planar and Fibre Optical Waveguides.* Mir (1980).
6. S. G. Krivoshlykov and I. N. Sisakyan. *Opt. Quantum Elec.* **11**, 393 (1979).
7. S. G. Krivoshlykov and I. N. Sisakyan. *Kvantovaya Elektronika* **7**, No. 3 (1980).
8. M. Adams. *Introduction to the Theory of Optical Waveguides.* Mir (1984).
9. H. Bateman and Erdelyi. *Higher Transcendental Functions*, Vol. 2. Nauka, Moscow (1974).
10. M. A. Golub, S. V. Karpeev, S. G. Krivoshlykov, A. M. Prokhorov, I. N. Sisakyan and V. A. Soifer. *Kvantovaya Elektronika* **10**, 1700 (1983).
11. M. A. Golub, S. V. Karpeev, S. G. Krivoshlykov, A. M. Prokhorov, I. N. Sisakyan and V. A. Soifer. *Kvantovaya Elektronika* **11**, 1869 (1984).