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NONLINEAR EVOLUTION OF DIVERSE PULSE SHAPES IN AN OPTICAL FIBRE

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Abstract—The evolution of the complex envelope of a family of single and pair pulses having the form of truncated elliptic cosines $\mathrm{cn}(\tau, k)$ with different k values $0 < k \leqslant 1$, including pulses corresponding to solitons (for which $k = 1$), has been studied by means of a numerical solution of the nonlinear Shcroedinger equation describing the propagation of an intensive localized wave field in a waveguide with cubic polarization characteristics. Analysis of the nonlinear interaction of pair pulses enables one to estimate the limiting transmission rates of digital signals and to work out recommendations for selection of the optimal pulse form.

Considerable attention has been devoted in recent years to controlling the transmission of intense ultrashort pulses in optical fibres, in the context of the latter's possible use for extremely fast digital signal transmission [1, 2]. The evolution of a normalized complex envelope function $\psi(\eta, \tau)$ of a localized wave field in an optical fibre with cubic polarization characteristics and negligibly small attenuation is described by the nonlinear Schroedinger equation

$$\mathrm{i}\frac{\partial \psi}{\partial \eta} + \frac{\partial^2 \psi}{\partial \tau^2} + \chi|\psi|^2\psi = 0 \tag{1}$$

derived by neglecting dispersion beyond second order, as well as a whole series of secondary nonlinear phenomena associated with the dependence of the group velocity on intensity, the generation of higher order harmonics, etc. The initial condition $\psi(0, \tau) = \psi_0(\tau)$ is determined by the pulse entering the light guide, χ is a nonlinearity parameter depending on medium properties and on the power of the incident pulse (the + sign of χ indicates anomalous dispersion), and η, τ are normalized coordinates related to the longitudinal propagation coordinate z and the time t by

$$\eta = z/L_0, \qquad \tau = (t - z/v_0)/T_0.$$

Here v_0 is the group velocity, and L_0, T_0 are characteristic length and time scales (for example, the "dispersion length" and the halfwidth of the initial pulse) [2].

In addition to well-known soliton solutions of the nonlinear Schroedinger equation. having an envelope $|\psi(\eta, \tau)|^2 = ch^{-2}(\sqrt{\chi/2}\tau)$ many other pulse-stable regimes are possible, distinguished from solitons by the large number of degrees of freedom, and consequently more suitable for transmitting information [2]. The transmission of pulses in the shape of a truncated elliptic cosine is of special interest, since a function like

$$\psi(\eta, \tau) = \mathrm{cn}(\tau/\Delta, k)\mathrm{e}^{\mathrm{i}a\eta} \tag{2}$$

represents a stationary solution of the nonlinear Schroedinger equation [3]. The parameters Δ and a of this function are given by

$$\Delta = \sqrt{\frac{2k^2}{\chi}}, \qquad a = \chi\left(1 - \frac{1}{2k^2}\right) \tag{3}$$

and equation (2) represents a family of envelopes depending on the parameter $0 < k \leqslant 1$, of which $k = 1$ corresponds to the soliton.

The transition to truncated functions is associated with the requirement of having finite pulses in order to transmit digital signals. Thus

$$\psi_0(\tau) = \begin{cases} \mathrm{cn}(\tau/\tau_0, k), & |\tau| \leqslant K \\ 0, & |\tau| > K, \end{cases} \tag{4}$$

where K is the quarter period. Unlike the periodic oscillation of (2), the pulse in (4) is of course

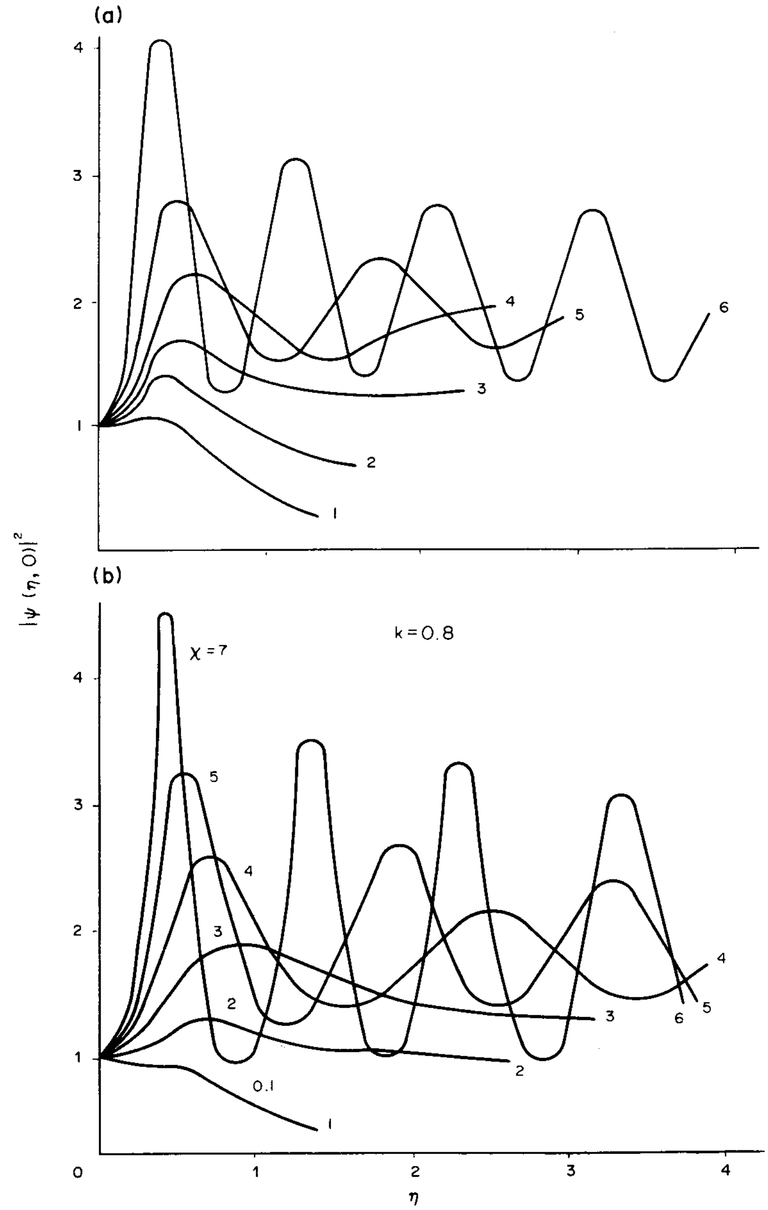

Fig. 1. (a) $k = 0.1$ and 1: $\chi = 0.1$; 2: $\chi = 2$; 3: $\chi = 3$; 4: $\chi = 4$; 5: $\chi = 6$; 6: $\chi = 7$. (b) $k = 0.8$ and 1: $\chi = 0.1$; 2: $\chi = 2$; 3: $\chi = 3$; 4: $\chi = 4$; 5: $\chi = 5$; 6: $\chi = 7$.

no longer a solution of the nonlinear Schroedinger equation (1), and the pulse form is not preserved when transmitted through the guide. The evolution of such pulses, obtained by digital modulation, will next be discussed.

Figure 1a,b illustrates the change in the pulse intensity maximum $|\psi(\eta, 0)|^2$ along the fibre as a function of χ. As in other, similar problems [2], one may choose a subcritical or a supercritical evolution regime. In the subcritical regime ($\chi < \chi_{cr}$) $|\psi|^2$ increases weakly at first and then drops off monotonically as η increases, while in the weakly supercritical domain ($\chi \gtrsim \chi_{cr}$) the pulse intensity maximum reaches a plateau value. On subsequent further increase of the incident pulse power $|\psi|^2$ evolves in oscillatory fashion, approaching a plateau value at infinity whose value is the larger, the greater the value of the nonlinearity parameter. The period of oscillation decreases with increasing χ. The change of the pulse width is inversely proportional to the intensity maximum.

A comparison of Fig. 1a and b shows that an increase in the initial pulse shape parameter k is accompanied by a decrease in χ_{cr} and an increase in the period and amplitude of the intensity maximum oscillations.

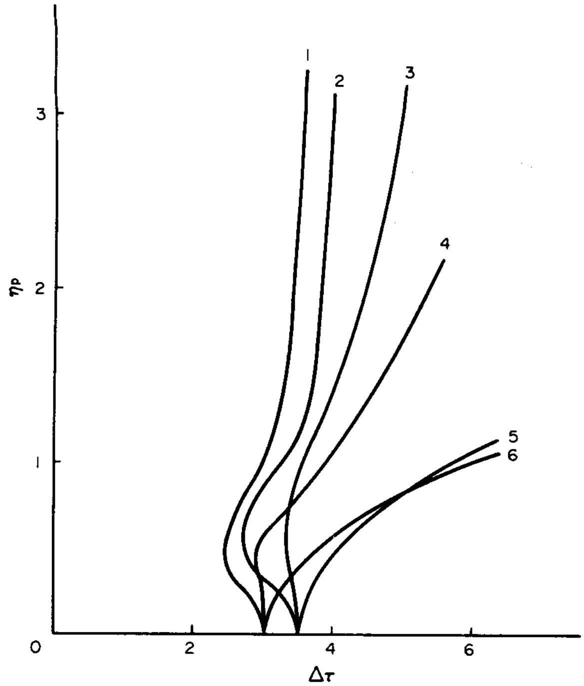

Fig. 2. 1: $\chi = 5, k = 0.1$; 2: $\chi = 5, k = 0.8$; 3: $\chi = 2, k = 0.8$; 4: $\chi = 2, k = 0.1$; 5: $\chi = 0.1, k = 0.8$; 6: $\chi = 0.1, k = 0.1$.

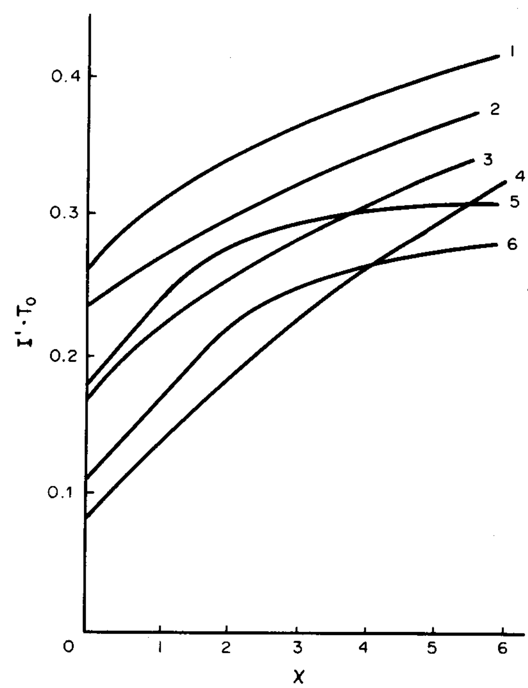

Fig. 3. 1: $k = 0.1, \eta = 0.5$; 2: $k = 0.8, \eta = 0.5$; 3: $k = 0.1, \eta = 1$; 4: $k = 0.1, \eta = 2$; 5: $k = 0.8, \eta = 1$; 6: $k = 0.8, \eta = 2$.

In order to gain an idea of the largest attainable digital signal transmission rate through an optical fibre channel, we studied the interaction of two pulses of the form $\psi_0(\tau) = \mathrm{cn}\left(\frac{\tau - \Delta\tau/2}{\tau_0}, k\right) + \mathrm{cn}\left(\frac{\tau + \Delta\tau/2}{\tau_0}, k\right)$. Figure 2 shows graphs of the limiting normalized guide length η_p at which the pulses are still resolvable, plotted as a function of the interval $\Delta\tau$ for different values of the nonlinearity parameter χ and pulse shape k, The reason that $\eta_p(\Delta\tau)$ is not single-valued at large χ is that at large nonlinearity χ the initially "unresolved" pulses are narrowed somewhat in the evolution process, so that they become resolved. The subsequent broadening and mutual interactions of the pulses result in a loss of pulse resolution.

By using these curves one can estimate the maximum pulse repetition rate at the guide entrance which allows discrimination of subsequent pulses at the exit pickup. For example, to transfer information over a single dispersion length by means of signals appropriate to an initial shape parameter $k = 0.8$, one requires them to be spaced at 5.75 halfwidths if $\chi = 0.1$ and 3.6 halfwidths if $\chi = 2$. The minimum attainable pulse separation at specified values of χ and k will be a function of the guide length η, i.e. $\Delta\tau_{min} = f(\eta)$.

If signal readout is independent and noise is negligible, then the maximum information transfer rate (in bits/sec) is

$$I'(\eta) = \log_2 m / T_0 f(\eta),$$

where m is the code base. In practice, of course, m cannot be arbitrarily large and its value (and with it, I') is noise-limited and is given by information theory. Since $f(\eta)$ grows as T_0 decreases, I' too cannot increase without limit.

Figure 3 shows the dependence of the normalized rate $I' \cdot T_0$ on the nonlinearity parameter χ using binary signals ($m = 2$), for various values of the guide length and the shape parameter k. The graphs show that, all things being equal, increasing the input power enhances the information transmission rate.

The shorter the transmission distance η, the smaller the value of k necessary to achieve the maximum rate. For large distance the optimal pulses for maximum rate are solitons, corresponding to $k = 1$.

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