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PHASE OPTICAL ELEMENTS WITH ARBITRARILY SPECIFIED DIRECTIVITY DIAGRAMS

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Abstract—Consideration is given to controlling the directivity diagram of raster phase optical elements. The feasibility of approximate solutions is discussed, together with their stability and realizability in various spectral ranges. Experimental results for IR and SHF ranges are described. The requisite real-time algorithms and graphic tools are discussed which could further develop this investigation.

INTRODUCTION

Many present-day practical problems of modern optics require the construction of directivity diagrams (DD) for synthetic optical elements. Among these are problems in holography, integrated optics, optical information processing, focusing of coherent radiation and optical tomography. Significant progress in the solution of such problems has been achieved in antenna theory [1].

The problem of constructing directivity diagrams is no less important in optical problems, and its application is fairly wide ranging [2]. An example purely of an optical element with a given directivity diagram is a phase diffraction lattice with a given energy distribution over the various orders. By combining such an element with other synthesized and traditional phase optical elements a series of interesting devices may be obtained, such as beam expanders, multifocus lenses and image multipliers, multi-channel filtering systems, kinoform, focusers, and so on.

The present work is devoted to a one-dimensional IR phase element, but in addition leads to a series of results relating to VHF elements and kinoforms, both in one and two dimensions.

We prove the solubility of the inverse problem of constructing elements with given DD utilizing a minimal set of free parameters, and discuss the accuracy of the solutions we obtain.

Several hundred model calculations on the computer formed an important part of the work. A phase diffraction lattice served as the experimental object, since it facilitated a simple and accurate measurement of the directivity diagrams, without having to invoke complex specialized apparatus.

BASIC DESIGN EQUATIONS

We assume that a plane wavefront of coherent monochromatic radiation is normally incident on the surface of a phase optical element. We denote by α the angle of diffraction, $u = (\sin\alpha)/\lambda$ the spectral coordinate, and λ the wavelength. Then the light intensity diffracted into the u direction is given by

$$I(u) = |F(u)|^2 |\sin(\pi N_0 u d)/\sin(\pi u d)|^2, \tag{1}$$

where N_0 is the number of periods and d is the width of a single period. Here $F(u)$ designates (to within a constant) the Fourier transform of the phase function with period $p(x)$ [3]:

$$F(u) = \int_0^d \exp(\mathrm{i}p(x) - 2\pi \mathrm{i}ux)\,\mathrm{d}x. \tag{2}$$

Since the elements which are designed are normally produced as flat optics elements [4], the most natural profile turns out to be a step function (*cf.* Fig. 1), for which the period is shared among N elements of dimension δ: $N\delta = d$. For such a profile Eq. (2) leads to a discrete Fourier transformation

$$\begin{aligned} F(u) &= \text{const} \times \operatorname{sinc}(u\delta) \sum_{k=0}^{N-1} \exp(\mathrm{i}p_k - 2\pi \mathrm{i}ku), \\ F_n = F(u_n) &= \text{const} \times \operatorname{sinc}(n/N) \sum_{k=0}^{N-1} \exp(\mathrm{i}p_k - 2\pi \mathrm{i}kn/N). \end{aligned} \tag{3}$$

Here $u_n = n/d$ are the principal directions (the maxima) of the lattice, and $|u_n| < 1/\lambda$.

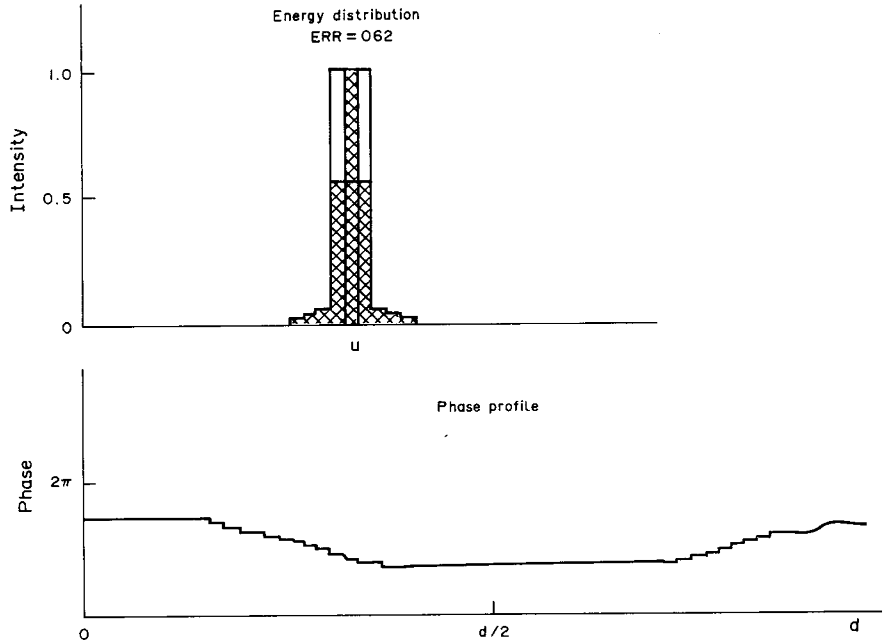

Fig. 1. DD with energy maxima in three orders.

Equation (3) enables one to calculate the field at the principal maxima of the lattice by using FFT (fast Fourier transforms), essential in producing a simple and effective modelling and solution of this problem on a computer.

Equations (1) and (3) determine the field over the whole range of u. The problem thus becomes one of determining the phases p_k that give the best approximation to $|F(u)|$, i.e. to the DD of the problem. We need only consider F_n, since $F(u)$ is reconstructed from the F_n in an obvious manner (via the sampling theorem).

COMPUTING ALGORITHM

We choose the following quantity to express the proximity of the directivity diagram to its desired value

$$\varepsilon = \|\tilde{a} - \langle \tilde{a} \,|\, \tilde{b} \rangle \tilde{b}\|^2, \tag{4}$$

where $\tilde{a} = a/\|a\|$, $\tilde{b} = b/\|b\|$ are the normalized vectors corresponding to $a = \{a_n\}$, $b = \{b_n\}$, a_n are the specified quantities $F_n = |F(u_n)|$, b_n are the computed counterparts, and $\|a\|^2 = \sum_n a_n^2$, $\langle a \,|\, b \rangle = \sum_n a_n b_n$.

Thus ε has the physical meaning of the energy of the compensating field that has to be added to the calculated value in order to obtain the required accuracy for the directivity diagram (in relative energy units the given field is taken to be equal to unity).

A Hertzberg–Saxton-type iterational algorithm [5] was the tool of choice for calculating the directivity diagram, using various spatial and frequency (spectral) restrictions: amplitude correction, phase quantization, and diffraction correction on the raster. The algorithm was based on standard FFT methods. All programs were written in FORTRAN, and operated in the interactive mode. (Since the CPU time is very small, the whole complex computation for a single element engaged just a few minutes of minicomputer display time.) The program also provided for calculating various aberration corrections, both phase and amplitude, of the optical elements obtained, input/output terminals for constructing the photomoulds required in the manufacturing stage, as well as graphics for a clearer presentation of the computational results.

ANGULAR ALIGNMENT OF DIRECTIVITY DIAGRAM

One of the most interesting problems is the alignment of the DD at a given angle. The type of DD indicated by the heavy line in Fig. 2 may, for example, be used to construct a phase diffraction

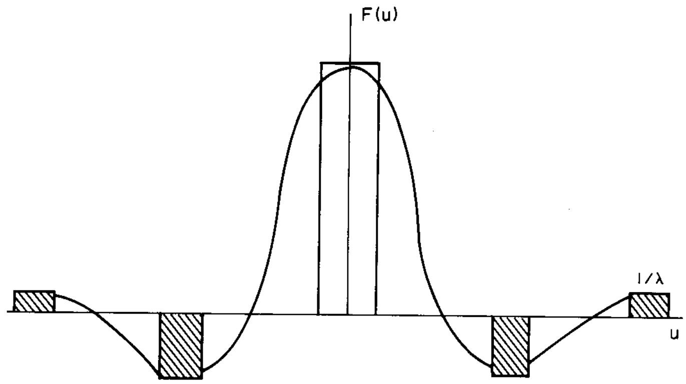

Fig. 2. Ideal rectangular DD (heavy line) and side lobes of a real DD (hatched regions).

lattice splitting a plane wave front (a parallel beam) into a bundle of identical plane fronts (beams), to construct a multifocus lens with linear or rectangular matrix of foci (having identical energy distribution at each focus), or elements focusing coherent radiation into segments, or rectangles.

The possibility of solving this type of problem was first demonstrated in [6].

For calculating the phase function with the requisite DD one may take

$$|F_n| = 1 \qquad |n| \leqslant L$$
$$|F_n| = 0 \qquad |n| > L.$$

The quantity L specifies the DD alignment angle. For an N-element raster calculation we must have $L \leqslant N/2$. The presence of a raster is manifested in the appearance of side lobes:

$$F(n/d) = \operatorname{sinc}(n/N) \sum_{k=0}^{N-1} \exp(\mathrm{i}p_k - 2\pi \mathrm{i}kn/N).$$

The periodic function within the summation is modulated by the factor $\operatorname{sinc}(n/N)$ (the side lobes are hatched in Fig. 2). Although one calculates N amplitudes F_n, physically their number is around $2d/\lambda$, so that only the central part (half-width) of the principal lobe is calculated (i.e. the zones $|n| < N = 2$). In order to suppress the DD outside the specified angle it is better to use $L \leqslant N/4$ when the energy bursts may appear only in the side lobes. Now, since $N\delta = d$, the number of side lobes will be determined by the ratio of δ and λ. There are no sidelobes if $\delta < \lambda/2$, and in this case one may compute a purely rectangular DD.

However, there are two further factors distorting a real DD which must be made rectangular.

The first factor is the analogue of the Gibbs phenomenon, and is expressed by the fact that if the DD is given as a pure rectangle (u with a sharp cutoff at the edges), then within the working region, and very near its boundary, uncontrolled oscillations appear whose amplitudes cannot be reduced by computational means (Fig. 3). One can suppress this effect significantly by first specifying the desired DD to be trapezoidal rather than rectangular, i.e. by having a smooth cut-off at the edges. The DD obtained thereby has a larger root-mean-square deviation from the rectangular than the existing oscillations. Nevertheless, the accuracy is considerably higher if frequencies are restricted to the region where the DD amplitude is a constant. It turns out that for good alignment (within 1–2%) a certain energy "cost" in losses is inevitable (usually up to 5%).

The second factor is the decrease in diffractive effectiveness with spatial frequency, arising from the DD of a single raster element. For an exactly step-like profile (*cf.* Fig. 1) this effect is accounted for by the factor $\operatorname{sinc}(n/N)$, but for real elements the form of the raster profile can never be strictly rectangular, so that the actual decrease in diffraction effectiveness is small (*cf.* Fig. 4).

This effect may be totally compensated at the boundaries of the DD operating zone if the amplitudes employed in the calculation are the corrected values obtained from the experimental curve (*cf.* Fig. 4). Note that the accuracy with which one can measure the deviation of the real curve from the sinc-factor will govern the accuracy of the correction itself. This statement is valid for calculating the DD of any form (not just rectangular). The effect discussed has a useful

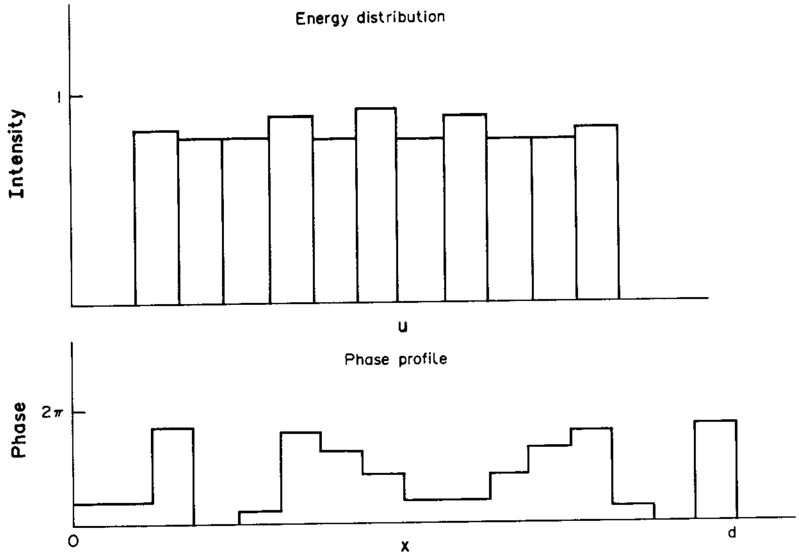

Fig. 3. Amplitude oscillations within a rectangular DD.

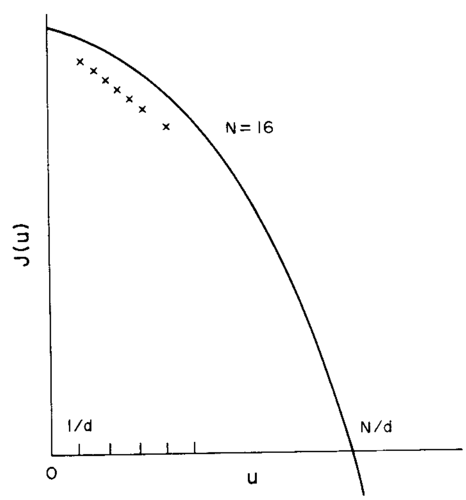

Fig. 4. Influence of raster form on real DD.

aspect—because of the change in relief profile form on the raster element, ensuring a sharper drop in the experimental curve (*cf.* Fig. 4), one may effectively suppress the side lobes, preserving at the same time the programmed correction of DD form in the central zone (the principal lobe).

ELEMENTS WITH ASYMMETRIC AND COMPLEX DIRECTIVITY DIAGRAMS

For almost any F_n distribution (i.e. DD) one can find the phase p_k, reproducing it with acceptable accuracy. It is hard to predict the degree of precision in advance. One can only say that, as a rule, it improves with the number of nonzero F_n. We shall only cite two examples from many. The first has unequal spacing between the active orders; in the second the DD has a complex asymmetry.

The first example is shown in Fig. 5 which has four active (i.e. nonzero) orders: $+5$, $+3$, -3, -5, with $N = 16$. The solution is quite accurate, $\varepsilon < 0.0001$.

The second example is shown in Fig. 6, where six orders are active with various energies, $N = 32$, and the accuracy is $\varepsilon = 0.05$.

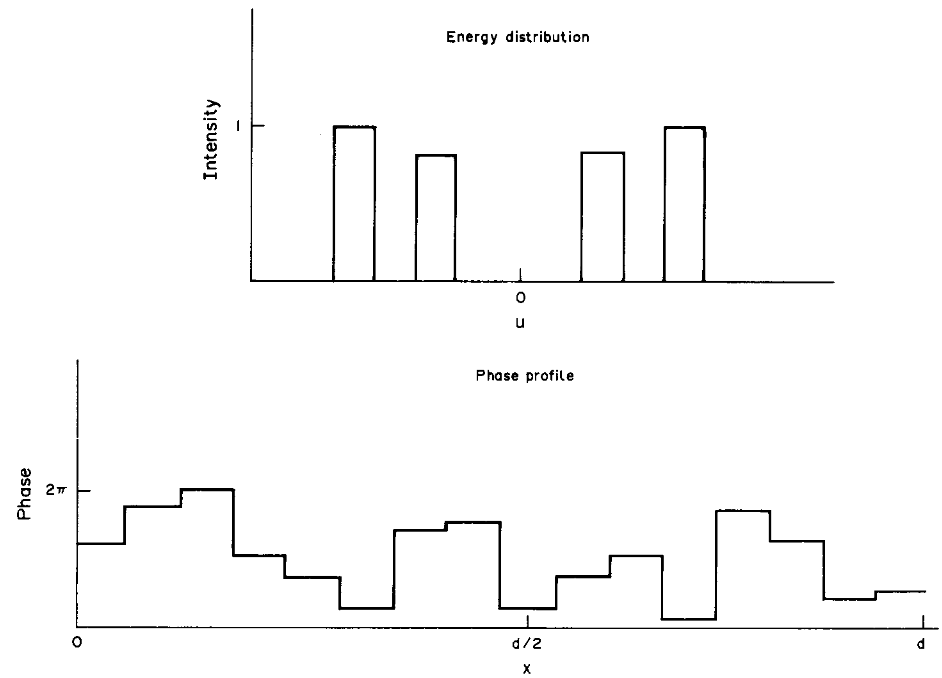

Fig. 5. DD with unequal steps between active orders.

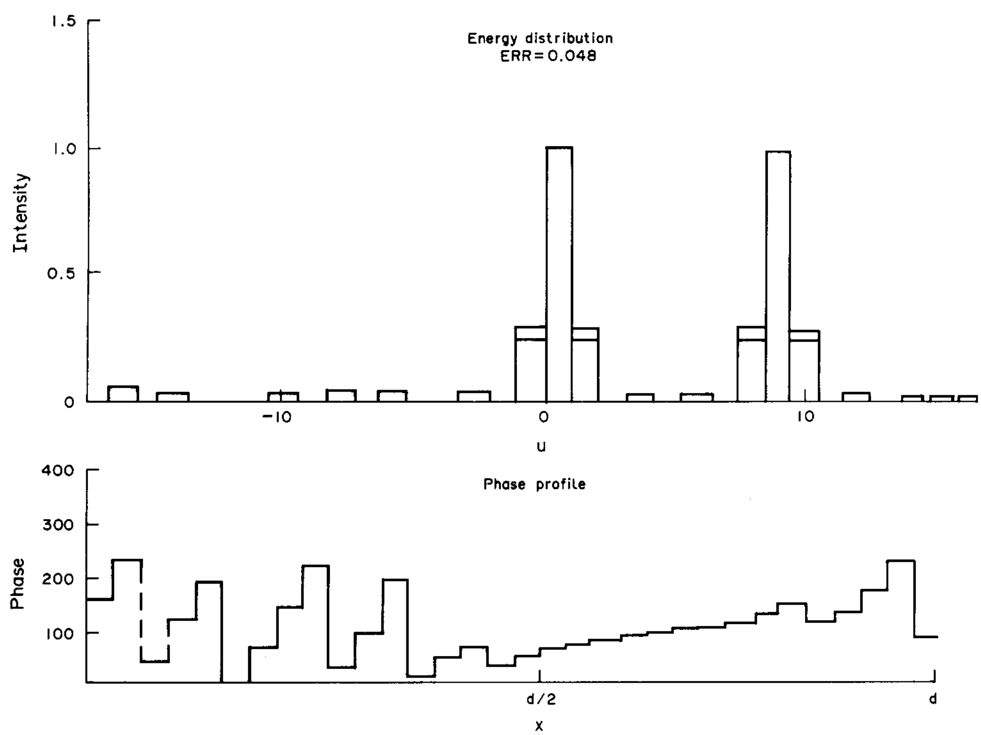

Fig. 6. Complex asymmetric DD form.

TWO-DIMENSIONAL ELEMENTS

If the DD factorizes as $F(u, v) = F_1(u)F_2(v)$, where u and v are spectral coordinates corresponding to x, y one may effect the search for the phase function that gives the requisite DD independently of x and y. The computed phase functions are combined to give the desired two-dimensional phase function. As has been proved experimentally and by computer modelling, even in the simplest variants the two-dimensional case makes higher precision demands both of the solutions, as well as of the manufacturing of the elements. The main reason for this is the crossed coherent interference of the additional diffracted wavefronts formed—because of inaccurate design and technology, not only for each coordinate but also among the coordinates themselves.

In the IR domain (at 10.6 μm) elements with quadratic, cellular and quasilinear (segmented) DD have been produced. The geometrical DD dimensions were maintained accurately enough, but within the DD appreciable intensity oscillations were observed, caused by effects discussed above.

THE EFFECT OF ERRORS IN SETTING THE DD PHASE

The strongest influence on the DD form of a phase optical element is due to the technical faults arising in imperfect element production processes. The general error common to all technologies is phase clipping (multiplying by a factor close to unity) and its nonlinear recording. Let f denote the phase, $0 \leqslant f \leqslant 1$, $f \to (1+c)f$ the clipping ($|c| \ll 1$), $f \to t(f)$ the nonlinearity of phase transfer, where $t(f)$ is a monotonic function, $t(0)=0$, $t(1)=1$. Both these errors may be united into a single form:

$$f \to (1+c)t(f) = r(f).$$

We then obtain for the transmission function of the element

$$\exp(\mathrm{i}r(f)) = \sum_{n=\infty}^{\infty} c_n \exp(\mathrm{i}nf), \qquad \sum_n |c_n|^2 = 1.$$

If phase transfer is ideal (perfect technology), $c = 1$, $c_n = 0$ for the remaining ns. In a real situation $|c_n|^2$ is the energy of the additionally arising wavefronts with multiple phase. For carrier elements (holograms) this energy does not pass into active diffraction orders, whereas for elements with lenses, the energy converges to the positive (real or imaginary) focus and is equal to F/n (F is the focal distance of the supplementary lens). In the latter case this energy may be measured directly, which enables one to determine the real phase transfer function $r(f)$ and with it the corresponding correction to the calculation. If this is not feasible, one may choose as the nonlinearity model,

$$t(f) = f(1-a-b) + af^2 + bf^2; \qquad |a| \ll 1, |b| \ll 1$$

and construct a table of values of $|c_1|^2$ for small a, b.

Clipping is best studied on concrete examples. Below we present a calculation for the element whose DD is shown in Fig. 5. For an element whose intrinsic error is small, the clipping error changes in the following fashion:

$1+c$	0.7	0.8	0.9	0.95	0.98	1.00	1.01	1.05	1.1
ε	0.23	0.11	0.03	0.008	0.001	0.000	0.000	0.01	0.3

EXPERIMENTS WITH ELEMENTS IN THE IR DOMAIN

In the experimental study of the DD of raster-type phase optical elements the relief was formed by sequential deposition in vacuum of thin layers of aluminium and copper on gelatine [4]. The photo mould (multigraded masks for duplicating the gelatine) was a raster scanning device outlet on the photographic film (P-1700). The elements were investigated experimentally. They were placed in a beam of single-mode 10.6 μm coherent radiation. Since in the course of the computation the phase function of the lens (of focal length $F = 400$ mm) was added to the phase function of the element, a Fraunhofer diffraction pattern was observed in the focal plane of this lens, consisting of a set of points (the principal maxima) for a lattice, rectangle, segment or similar, for an aperiodic element (a kinoform).

The energy measurements at the maxima were performed by means of a calorimeter. In addition the thermal effect of the radiation was observed on paper, plexiglass and other materials. The calorimetric measurements were estimated to be 8–10% accurate.

We measured the total energy of the incident radiation, the energy at the principal maxima of the lattice and at the positive multiple focal planes arising due to the nonlinearity of phase detection. As a rule, because of diffusional scattering, the existence of side lobes, and the significant nonlinear phase detection, the total energy $|c_1|^2$ at the principal focal plane constituted some 40–50% of the beam energy. The quantity $|c_2|^2$ was around 15%, and $|c_0|^2 \sim 10\%$, showing that the nonlinearity was significant. The accuracy of the measurement was not sufficient to reconstruct the function $r(f)$ to acceptable accuracy and, because of the unstable properties of the photomaterials and of the processing, $r(f)$ was very unstable. Through interference microscope measurements the clipping (the inaccuracy in relief depth control) was estimated around 10%, though it was quite nonuniform over the width of the elements.

At the same time the form of the DD at the principal focal plane for given calorimetric measurements practically coincided with the specified form, since it was essentially only influenced by the clipping, which was estimated to cause a DD form distortion of only $\varepsilon = 0.03$, undetectable visually or calorimetrically.

In order to measure the decrease of the diffraction effectiveness (DE) as a function of frequency (*cf.* Fig. 4) on a real raster, a lattice with $N = 32$ raster elements was produced and a_n was arranged to be unity for $|n| < 6$. The usual $\{a_n\}$ corrections were not carried out in this process. The data as presented in the following table bear out the assumption that the smoothing technique for the raster profile exerts a significant influence on the variation of DE decay with spatial frequency and with the side lobe size.

	0	1	2	3	4	5	6
Theory (a_n^2)	1	0.99	0.95	0.89	0.81	0.71	0.61
Expt (b_n^2)	1	0.96	0.87	0.77	0.66	0.62	0.57

EXPERIMENTS WITH ELEMENTS IN THE VHF DOMAIN

Numerically controlled machines were used to produce elements in the 3–5 mm wavelength range from plexiglass. The lens and diffracting lattice were manufactured from plexiglass separately (this was dictated by cutting technology requirements), with a refractive index $n = 1.6$ and width around 15 mm. The element size was approximately 500 × 500 mm, the focal length of the lens was of the order of 1000 mm. Three phase diffracting lattices were produced: one with suppressed zero order, one with the DD aligned at specified angle, and one for spectroscopy in the 3–5 mm wavelength interval. The results were monitored by means of a radio sighting device.

In this range the lattice diffracted, in zero order, not more than 20% of the energy (at the edges of the interval), the rest was divided equally at the boundary of the main lobe in the +1 and −1 orders. We therefore have a two-fold control in wavelength measurement; namely u, in terms of the geometrical deviation of the first order from zero; and by comparison of intensities.

The accuracy with which a relief depth of 0.1 mm at VHF may be realized corresponded to specifying the phase to within $\lambda/60$, which is an indication of the high optical quality of the elements manufactured, and their possible use in display schemes. The focal spot and line dimensions were practically the diffraction-limited values.

CONCLUSION

The method we have presented allows rapid construction of directivity diagrams for raster phase optical elements with limited computational resources. It is capable of being applied to real-time problems by using, for example, reversible holographic media.

The results that were obtained are not, as a rule, unique, and may be optimized by means of complex (and considerably more cumbersome) computational methods.

Practical studies carried out with the present method for a wide range of problems promise to lead to its application, when suitably modified, to integrated optics problems as well, in terms of the ability to control the properties of distributed inverse relations, the structure of the coherent field in crimped wave guides, etc. Finally, several examples from the field of biology (diffractive dielectric structures forming patterns on the wings of some butterfly species, diffractive masking structures on the scales of certain fish) bear witness that suitable control of directivity diagrams (especially over a broad spectral range) enables one to solve diverse complex problems.

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