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SPATIAL FILTRATION OF COHERENT RADIATOR HARMONICS

A. S. MARKIN and V. B. STUDENOV

Abstract—For optical harmonics of any integer (and fractional) order, a spatial filtration technique is considered making use of the zone plate dispersion properties. A method is suggested for estimating the filtration effectiveness as a function of filter parameters and the set of harmonics. Experimental data are presented on filtration of the second and third harmonics of a non-single-mode laser.

Selection of vacuum ultraviolet (VUV) harmonics from the spectrum of laser radiation harmonics was achieved in [1, 2] by means of a spatial filter (SF) consisting of a zone plate (ZP) with a specified number of zones and diaphragms situated in the ZP principal focus, chosen for the wavelength of the particular harmonic. It was shown that for integral harmonics the condition for optimal filtering could be guaranteed by choosing the appropriate number of ZP zones. The method can also be employed to select higher order harmonics in sources of VUV coherent radiation with cascade frequency multiplication (*cf.* [3, 4]). Here traditional filtering methods based on absorption filters, prisms and diffracting lattices are useless, on account of the strong VUV absorption of various media, and of the impossibility of working with focused beams.

In the present work the general nature of the filtering effectivity is examined as a function of SF parameters, and of the wavelengths in the original discrete spectrum.

Let us suppose that two plane monochromatic waves λ and λ' are incident on the ZP, λ being the selected wave. In order to calculate the condition for optimal filtering we use the formula for the distribution of wave amplitudes near the ZP axis, when a plane monochromatic wave λ of unit amplitudes is normally incident upon it [5]:

$$A(x, r) = \sum_{m=0}^{N} I_0\left(\frac{2\pi}{\lambda} r \sin \gamma_m\right) \exp\left[i\left(\frac{2\pi}{\lambda} R_m + m\pi\right)\right]. \tag{1}$$

Here x is the coordinate on the ZP-axis; r is the distance from the ZP-axis; $R_m = \sqrt{x^2 + \rho_m^2}$; $\rho_m = \sqrt{m\lambda L}$ is the radius of the mth ZP zone; L is the principal focal length; $I_0(2\pi/\lambda \cdot r \sin \gamma_m)$ is the zero-order Bessel function; γ_m is the angle subtended by the mth zone radius at a specified point on the ZP-axis; N is the total number of ZP zones.

Formula (1) was obtained for a ZP with a first transparent zone and an odd number of zones, assuming the angles γ_m are all small. Then in the λ wave principal focal plane the amplitude distribution for the λ' wave has the form:

$$A'(L, r) = \exp\left(i \frac{2\pi}{\lambda'} L\right) \sum_{m=0}^{N} I_0\left(2\pi r \frac{\lambda}{\lambda'} \sqrt{\frac{m}{\lambda L}}\right) \times \exp\left[im\pi\left(1 + \frac{\lambda}{\lambda'}\right)\right]. \tag{2}$$

We introduce a dimensionless parameter $\chi = 1.22r/\delta R_0$, where $\delta R_0 = 0.61(\lambda L/N)^{1/2}$ is the radius of the principal focus of the wave in a cross-section that is perpendicular to the ZP axis. In this case (2) becomes,

$$A'(L, \chi) = \exp\left(i \frac{2\pi}{\lambda'} L\right) \sum_{m=0}^{N} I_0\left(\pi\chi \frac{\lambda}{\lambda'} \sqrt{\frac{m}{N}}\right) \times \exp\left[im\pi\left(1 + \frac{\lambda}{\lambda'}\right)\right], \tag{3}$$

while at the λ wave principal focal point

$$A'(L,0)=\exp\left(\mathrm{i}\frac{2\pi}{\lambda'}L\right)\sum_{m=0}^{N}\exp\left[im\pi\left(1+\frac{\lambda}{\lambda'}\right)\right]$$

$$=\exp\left(\mathrm{i}\frac{2\pi}{\lambda'}L\right)\frac{\exp\left[\mathrm{i}(N+1)\left(1+\frac{\lambda}{\lambda'}\right)\pi\right]-1}{\exp\left[\mathrm{i}\left(1+\frac{\lambda}{\lambda'}\right)\pi\right]-1}. \tag{4}$$

It follows from (3) that the amplitude distributions of the λ and λ' waves in the principal focal plane of the λ wave is determined solely by the ratio λ/λ', the number of ZP zones and the diameter of the filtering aperture. Expression (4) provides a rule for choosing the ZP zone. The filtering will obviously be most effective when the total number of ZP zones is such as to minimize $A'(L,0)$. If the ratio of λ and λ' is p/n, where p and n are integers, then the amplitude $A(L,0)$ vanishes whenever $N=2qn-1$ $(q=1,2,\ldots)$ for all $\lambda<\lambda'$, as well as for $\lambda>\lambda'$, excluding the cases when $\lambda/\lambda'\neq 2\xi-1$, where $\xi=1,2,3,\ldots$. If $\lambda/\lambda'=2\xi-1$, the principal λ-wave focus coincides with the $(\xi+1)$-order secondary focus of the λ' wave, as can be seen from (4), the amplitude of both waves at this point being equal to $(N+1)$. The transverse focal dimensions are different and are determined by (3). If the original radiation consists of discrete components associated with integer number ratios, then the amplitudes of all λ' waves, except the one to be selected $\lambda_{\min}$, will vanish at the principal focus of the $\lambda_{\min}$ wave whenever

$$N=2qs-1, \tag{5}$$

where s is the smallest common factor of the fraction $\lambda_{\min}/\lambda'$.

The proposed SF may therefore be used to select fractional order harmonics which may arise, for example, in the interaction of powerful laser radiation with plasma [6].

Equation (3) was used as the starting point for a computer calculation of the optimal effectivity for filtering λ_k waves from a spectrum of λ_j waves, where $\lambda_j=\lambda/j$, $j=1,2,3,\ldots,k$, valid for both integers as well as fractional harmonics with identical numerators. According to (5), the filtering of the λ_k waves will proceed most effectively when the total number of ZP zones satisfies $N=2qk-1$. Since all waves arriving at the SF have unit amplitudes, the filtering effectivity η_{kj} of the λ_k waves from among the λ_j waves will be determined by the ratio of the corresponding energy currents of these waves across the SP diaphragm, i.e.

$$\eta_{kj}(\chi_0)=\frac{\int_0^{\chi_0}A_k^2(\chi)\chi\,\mathrm{d}\chi}{\int_0^{\chi_0}A_j^2(\chi)\chi\,\mathrm{d}\chi},$$

where χ_0 is the diaphragm radius.

Calculations have shown that the filtering effectivity for $k\geqslant 3$ with zone plates having $N\sim 30$ is already in excess of 10^3 and grows with the number of selected harmonics. Figure 1 shows η_{kj}, with $k=3$ and $j=1,2$, as a function of SF diaphragm radius. Figure 2 shows the results for $k=4$ and $j=1,2,3$.

We infer from these figures that the filtering effectivity of higher harmonics increases dramatically as the diaphragm diameter is decreased. The calculation also showed that for $0\leqslant\chi\leqslant 1.2$ the distribution of the jth squared amplitude, $A_j^2(\chi,N)$, in the SF diaphragm's plane is practically independent of the total number of zones when $N>30$. Hence the ratio of the filtering effectivities for two zone plates with N_1 and $N_2<30$ is given quite accurately by the relation

$$\frac{\eta_{kj}(\chi,N_1)}{\eta_{kj}(\chi,N_2)}=\left(\frac{N_1+1}{N_2+1}\right)^2. \tag{6}$$

Equation (6) relates the fundamental characteristics of various SFs, and enables one to calculate the filtering effectivity of any SF in terms of the known parameters of another one.

The experimental realization of the proposed filtering method has been carried out for the second and third harmonics of a neodymium laser, obtained by a cascade transformation of the fundamental width ($\lambda=1.064$ nm) through a KDP crystal. The fundamental and the two harmonics were guided

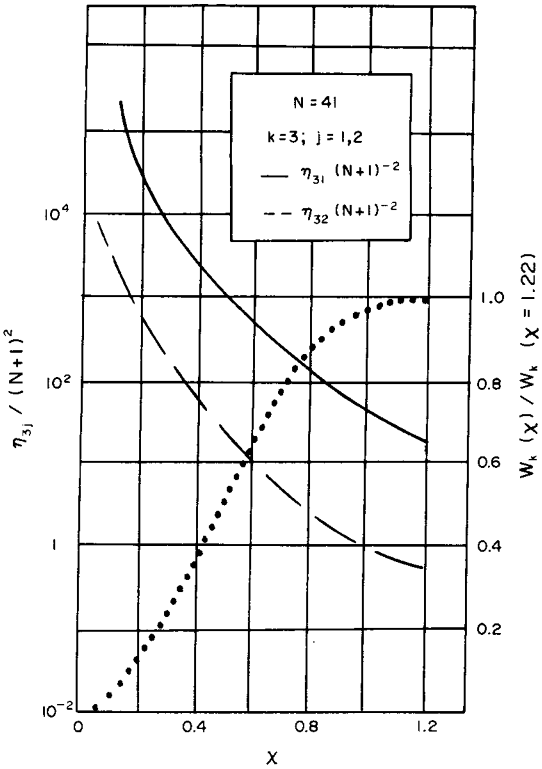

Fig. 1. Reduced effectivity η_3 and power W_3 for a selected third harmonic (from a spectrum of first- and second-order harmonics) as a function of diaphragm radius χ.

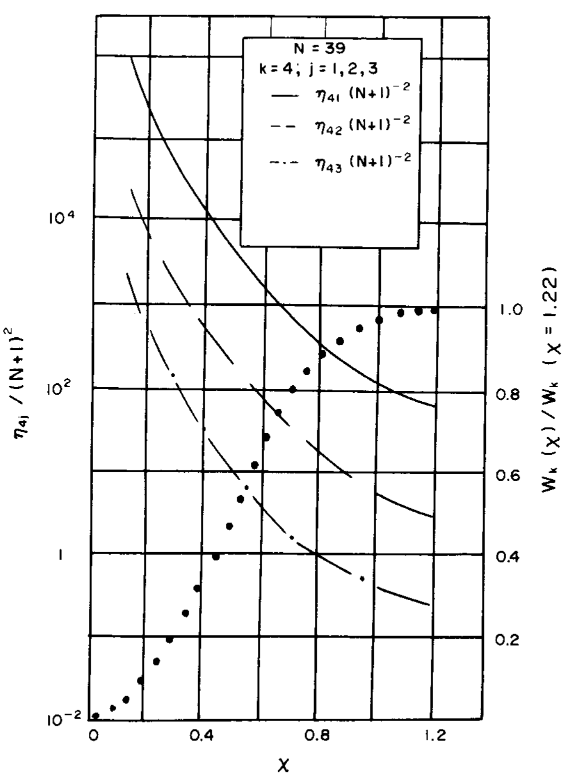

Fig. 2. Reduced effectivity η_4 and power W_4 of a fourth harmonic (generated from a spectrum of first, second and third orders) as a function of the diaphragm radius χ.

by a ZP with $N = 41$ manufactured by the photolithographic process. A diaphragm of 100 μm diameter was located at the principal focus of the ZP ($\lambda_3 = 1$ m) for the third-order harmonic. The values of the filtering effectivity obtained thereby, $\eta_{31} \sim 8 \times 10^3$ and $\eta_{32} \sim 3 \times 10^2$, are in good agreement with the calculated values $\sim 2 \times 10^4$ and $\sim 10^3$, respectively.

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