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WAVEFRONT FORMATION IN LONGITUDINALLY NONUNIFORM ASYMMETRIC GRADIENT WAVEGUIDES

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Abstract—A new refractive index reference profile is suggested for describing asymmetric gradient waveguides. The effect of regular longitudinal non-uniformities on the wavefront of beam in these waveguides is studied. Relations have been determined for estimating the field amplitude across the beam at the interface of two such waveguides. The possibility is demonstrated of determining medium parameters through the known characteristics of the primary and final beams.

The determination and formation of beam wavefronts is of interest in the investigation of the propagation of various sorts of radiation in artificial or natural waveguides.

If the refractive index of the medium n changes slowly on the scale of the wavelength λ (i.e. $\lambda \nabla n/n \ll 1$), then for harmonic fields radiation propagation is described by the scalar Helmholtz equation [1–3]:

$$\Delta\Psi = k^2 n^2 \Psi = 0, \tag{1}$$

where $k = 2\pi/\lambda$.

Equation (1) admits of an exact solution in quadratures only for a restricted number of $n(z)$—the reference refractive index functions—which were surveyed in [1].

Gradient waveguides with symmetric profiles are usually approximated by a parabolic refractive index distribution. In that case Eq. (1) is equivalent, in the paraxial approximation, to the Schroedinger equation for a variable frequency harmonic oscillator, which means it can be studied by well-developed quantum mechanical methods. For example, the coupling coefficients between the modes of the initial and final longitudinally uniform sections were determined [4] in a waveguide containing regular longitudinal nonuniformities. The expressions obtained there enable one to determine the beam wavefront at the waveguide exit, given that it is known at the entrance.

On the other hand, a knowledge of beam parameters in such a waveguide allows one to solve the inverse problem, namely, to determine the nonuniformity parameters of the medium. Thus, it was proposed to utilize experimentally obtained relative intensities [5] in order to determine the waveguide gradient parameter change and the transverse axial shift, while in [6] from measurements of the final width of the beam for various initial widths the variability of the waveguide's longitudinal nonuniformities could be quantified.

The analogous problem for asymmetric gradient waveguides has not yet been investigated, even though it is of interest, inasmuch as a series of gradient waveguides are characterized by explicit asymmetries. Such waveguides are most underwater sonic channels (USC) in deep and shallow oceans (for example, the USC of the Mediterranean sea [7]), as well as the ionospheric radio channel [2]. Asymmetry in the transverse distribution of the refractive index is observed also in diffusional integrated-optical lightguides, and in active waveguides generated in the transverse plane of heterojunction lasers [3].

In the present investigation of the formation of wavefronts in asymmetric waveguides it is proposed to adopt a new reference profile to account for the longitudinal nonuniformities of the gradient medium. On the basis of the suggested model profile the following questions are examined:

—The influence of longitudinal nonuniformity parameters on the radiation wavefront,
—The reduction of nonuniformity parameters to known parameters of the radiation.

For simplicity, we consider a two-dimensional waveguide and introduce a reference refractive index profile:

$$n^2(x;z) = n^2(x_0;z) - \omega^2(x^2 - x_0^2) - 2g\left(\frac{1}{x^2} - \frac{1}{x_0^2}\right) \quad x > 0, q \geqslant 0 \tag{2}$$

where x_0 is the coordinate of the waveguide axis, $n(x_0; z)$ is the refractive index along the axis, ω is the refractive index gradient, and g is the asymmetry parameter.

Equation (1), in the paraxial approximation for the profile (2), is equivalent to the Schroedinger equation for a single oscillator, so that quantum mechanical methods can be used to study it [8].

An arbitrary field $|\psi\rangle$ in the waveguide (2) may be represented by a model expansion in which the squared expansion coefficients $|\langle n|\psi\rangle|^2$ assign the field distribution among the modes. If the medium is uniform longitudinally, then in the paraxial approximation all modes propagate with the same group velocity (the propagation constant spectrum is equispaced [9]), and the initial phase front of the exciting beam periodically restores its own form. For points between those values of z for which the beam front is reestablished, knowledge of the medium parameters enables one to calculate the phase of the beam wavefront easily.

The presence of longitudinally nonuniform sections in the medium (2) induces a redistribution of energy among the modes, i.e. a change in the wave field amplitudes.

The authors have set up recursion relations and using them calculated the mode coupling coefficients for two butted asymmetric gradient waveguides. Given the wavefront in the first guide, its value in the second can be calculated by using the relations we have derived. The mode overlap integrals are given by the following:

$$T_m^n = \left(\frac{2\sqrt{\omega_1\omega_2}}{\omega_1+\omega_2}\right)^{\frac{1}{2}(a_1+a_2)+1}\left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{4}(a_1-a_2)}\left(\Gamma\left[\frac{a_1+m+1, a_2+n+1}{n+1, m+1}\right]\right)^{1/2}$$
$$\times\,\Gamma\left[\frac{\frac{1}{2}(a_1+a_2)+1}{a_1+1, a_2+1}\right]\cdot F_2\left(\frac{1}{2}(a_1+a_2)+1;\ -n,\ -m; a_1+1, a_2+1; \frac{2\omega_1}{\omega_1+\omega_2}, \frac{2\omega_2}{\omega_1+\omega_2}\right), \tag{3}$$

where $\Gamma(t)$ is the gamma function, and F_2 is the Appel function in two variables.

The coupling coefficients W_m^n are determined by the square moduli of the mode overlap integrals, and the coordinate of the waveguide axis is related to the parameter a by

$$x_{0i}^2 = \frac{a_i^2 - \frac{1}{4}}{\omega_i k}; \quad i = 1, 2. \tag{4}$$

The shift in the waveguide axis is also completely determined through the specified parameters:

$$\Delta_{x_0} = k^{-1/2}\left|\left(\frac{a_1^2-\frac{1}{4}}{\omega_1^2}\right)^{1/4} - \left(\frac{a_2^2-\frac{1}{4}}{\omega_2^2}\right)^{1/4}\right|. \tag{5}$$

In the following the effect of regular longitudinal changes in the gradient of the medium on the wavefront is studied for a constant value of a. The mode coupling coefficients and recurrence relations for calculating them are also obtained. The coupling coefficients are symmetric, $W_n^m = W_m^n$, and are completely fixed by the following numerical parameters: R the coefficient of super-barrier reflection [4], and a, the asymmetry parameter of the medium (2).

In the experimental determination of these numerical parameters the relative mode intensities $q(m, n) = W_n^m/W_0^0$ are helpful. Thus,

$$R = \left[\frac{2q(2,0)}{q(1,0)} - q(1,0)\right]^2, \tag{6}$$

$$a = \frac{q^2(1,0) - q(2,0)}{q(2,0) - q^2(1,0)/2}. \tag{7}$$

The parameter R may be estimated for two butted waveguides with $a = \text{const}$ and different gradient parameters ω_1, ω_2. Thus, $R^{1/2} = (\omega_1 - \omega_2)/\omega_1 + \omega_2) = \Delta\omega/2\omega$. Then the gradient parameter change can be estimated from (6).

Finally, we generalize the method of reconstructing the longitudinal waveguide nonuniformities, originally developed [6] for a parabolic index profile, to an asymmetric waveguide (2). The problem is reducible to the solution of the Gelfand–Levitan–Marchenko integral equation:

$$K(x, y) + F(x+y) + \int_x^{\infty} K(x, y)F(t+y)\,dt = 0, \quad x < y,$$

when $F(x) = \pi^{-1} \operatorname{Re} \int_0^\infty r(\omega) e^{-i\omega x} \, d\omega$ is the spectral function, and $r(\omega) = R^{1/2} \cdot e^{i(\delta_2 - \delta_1)}$ is the reflection coefficient.

In order to obtain the spectral function one must find the reflection coefficient for various widths of the beam incident on the nonuniform section. The reflection coefficient may be calculated using the expression [9]:

$$\langle x_+^2 \rangle = \frac{a+1}{k\omega_+(1-R)^2} \{(1+R)^2 + 4R\cos 2\delta_1 - 4R^{1/2}\cos\delta_1 \times [\cos(2\omega_+\xi + 2\delta_1 - \delta_2) + R\cos(2\omega_+\xi - \delta_2)]\}.$$

This relation connects the final width of the beam with the reflection coefficient. From here we determine the spectral function and solve the integral equation, extracting $K(x, y)$ and thereby reconstructing the unknown ω from $\omega^2(\xi) = -(d/dx)K(x, y)$.

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