

## Determination of the beam waist position for the spin-orbit interaction effect observation

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### Abstract

The spin angular momentum and the extrinsic orbital angular momentum of light are associated with the polarization of light and the light propagation trajectory, respectively. Those momenta are interdependent not only in an inhomogeneous or anisotropic medium but even in free space. This interaction is called the spin-orbit interaction of light. The effects of the spin-orbit interaction of light manifest themselves in a small transverse shift of the beam field longitudinal component from the beam propagation axis in the waist region under the circular polarization sign change. They can be observed both for Gaussian beams and for structured beams. The effects of the spin-orbit interaction of light should be taken into account when nanophotonics devices are created, but the detailed investigation of the effect had not been performed yet due to the low intensity noise image of the beam waist. Precise measurements of the focal waist centerline are needed to determine the transverse shift of the beam field longitudinal component of the asymmetric converging beam's waist under the circular polarization sign change. We propose methods for determining the transverse and longitudinal positions of the beam waist. Computer image processing methods made it possible to obtain the value of the beam waist's transverse position with an accuracy of 0.1  $\mu\text{m}$ . These methods will allow further testing of the shifts' theoretical predictions, the values of which are the order of 1  $\mu\text{m}$ . The results obtained can also be used for laser processing of materials by polarized light and precise positioning of the beam's focal spot at a surface.

**Keywords:** spin-orbit interaction of light, angular momentum of photon, waist position, image processing.

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### Introduction

There are three types of angular momentum of a photon: spin, intrinsic orbital, and extrinsic orbital [1]. The spin angular momentum is associated with the polarization of light [2], the light beam trajectory determines the extrinsic orbital angular momentum (OAM) [3], and the intrinsic orbital angular momentum is determined by the light field's structure of the beam [4, 5].

The spin and extrinsic orbital angular momenta are interdependent in an inhomogeneous or anisotropic medium. This interaction is commonly called the spin-orbit interaction of light, also known as the optical Spin Hall Effect (an optical analog of the Spin Hall Effect that occurs for charge carriers in solid-state systems) [6, 7]. In the most general case, the effect manifests itself in the transverse shift from the axis of propagation of the beam reflected from the two media boundary when the circular polarization sign changes. This shift is also known as the Imbert-Fedorov shift [8, 9, 10]. The transverse shifts at the core-cladding boundary result in the speckle pattern rotation of light propagating through the optical fiber under the circular polarization sign change, known as the Optical Magnus effect [11].

Such an effect can be observed in the free space, that is, regardless of the interaction of light with matter. Cur-

rently, this effect is also known as the Geometric Optical Spin Hall effect. For the first time, the effect was predicted theoretically [12], namely, that the transverse shift of the "center of gravity" of the longitudinal component of the asymmetrically converging Gaussian beam occurs in the waist's plane under the sign of circular polarization change. The displacement is small, and its value is the order of the beam waist radius. In [13] the first visual results of the experimental detection of the waist shift were presented. For the same light propagation scheme, a generalized theory [14] was proposed, which applies to a beam with an arbitrary intensity distribution in the cross-section. The value of the "center of gravity" shift was calculated in the general case. It was also shown that the same shift of the "center of gravity" takes place in all sections of the converging beam, and not only in the plane of the waist. But in practice, the observation is possible just in the focal plane because of the large size of the beam compared to the small value of the shift. The paraxial approximation limits the accuracy of theoretical calculations in [12, 14].

Besides, the splitting of a collimated linearly polarized Gaussian beam into two (with left and right circular polarization) beams was found in the plane inclined to the direction of light propagation in the free space. This was

theoretically demonstrated in [15]. The principal scheme of observation of this effect and experimental results were presented in [16, 17].

Further experimental investigations of the Geometric Spin Hall Effect were performed on the example of an asymmetric tightly focused beam. It consisted of two halves with the circular polarization of different signs in the cross-section [18]. In the waist's plane, the field's deformed distribution was observed due to the relative shifts of the electric energy density of the field's longitudinal components.

The intensity distribution of the  $z$ -component in the focal plane for a circularly polarized Gaussian beam is calculated [19]. The influence of the extrinsic orbital angular momentum on the spin angular momentum is experimentally demonstrated [20], that is, the influence of the trajectory of light propagation on its polarization in the free space. Several reviews have recently been published on spin-orbit interaction, generating light beams with angular moments, their registration, transformations, interactions with media, and applications of these phenomena [21–23].

The effects of the spin-orbit interaction of light in the free space are weak [24, 25]. Methods of high sensitivity and accuracy are required to observe the effects of the spin-orbit interaction of light in the free space experimentally. Besides, it is necessary to apply modern image processing methods, allowing estimating small beam displacements of the order of  $1 \mu\text{m}$ .

This work aims to propose the experimental methods for determining the Gaussian and the Bessel beam waist position and its displacement with high accuracy to observe the effects of the spin-orbital interaction of light subsequently. We should use low intensity and low contrast digital images with optical noise to solve this problem.

In general, the selection, recognition of objects, and determination of their parameters in the presence of various kinds of noises are some of the most complex and relevant tasks of image processing. In this case, the approximation problem arises since the original image has redundant information its use.

There are many algorithms for solving the approximation problem using the multivariable function [26], find edges [27–31], spline interpolation [32], including the use of adaptive triangular mesh [33, 34], and wavelets [35]. These algorithms are useful, for example, for compressing images or for recognizing their elements. However, to solve the problem, these methods are unsuitable, because the focal waist image has its features such as the lack of precise edges. It is not continuous but has a speckle structure. Therefore, an attempt to isolate the edges of the waist using differential operators leads to the appearance of impulse noises. Smoothing filters, tone correction, blurring, and contrasting algorithms cannot be used for the most precise determination of the waist parameters. Various types of threshold analysis, which are often used to highlight image elements and their edges, are also

not recommended because all of these image processing methods add an unacceptable error.

Due to these limitations, the method of approximating the image by a two-dimensional function of many parameters using the least-squares algorithm was chosen, which involves fitting parameters that minimize the sum of the squared distances between each data point and the function. Theoretical assumptions regarding the beam properties allowed us to choose the function type and the function parameters.

### 1. Method of determination of the Gaussian beam waist transversal position

To determine the transverse position of the longitudinal component of the electrical field ( $z$ -polarization) of the asymmetrically converging Gaussian beam in the waist region, we used the experimental setup shown in fig. 1. We observed the waist in the longitudinal direction ( $Oyz$  plane).

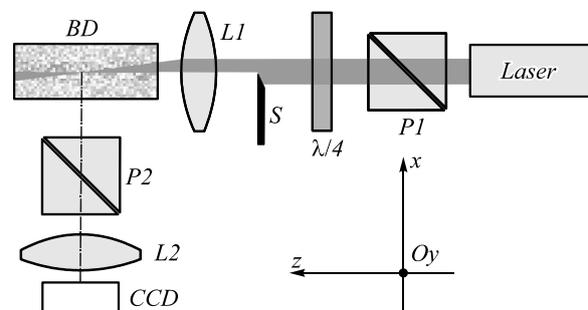


Fig. 1. The experimental setup for observation of the focal waist position of the Gaussian beam. P1, P2 are polarizers; S is shutter; L1, L2 are lenses; BD is beam diffuser

The He-Ne laser's radiation was passed through the beam expander, which enlarged the beam diameter by 2.54 times. The polarizer P1 and the phase plate  $\lambda/4$  were used to give circular polarization to the beam. Then, we got the beam's image in the transverse direction ( $Oxy$  plane) to determine its main parameters. The transverse image of the Gaussian beam is shown in fig. 2.

The intensity distribution of the beam cross-section shown in fig. 2 was approximated by the function:

$$f(x, y) = a \cdot \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) + f_0. \quad (1)$$

The approximation of the image is shown in fig. 3. The diameter  $d=2\sigma$  of the beam by criteria  $FW\epsilon^{-2}M$  was  $2.30 \pm 0.01 \text{ mm}$ .

The expanded laser beam with circular polarization fell on shutter S, which blocked half of the radiation. The lens L1 with the focal length of 20 mm formed the asymmetrically converged beam. The longitudinal image of the  $z$ -polarization of the beam is shown in fig. 4. The image enlarged with the lens system L2 was obtained using a CCD camera. We installed a polarizer P2 in front of the camera to select the  $z$ -polarization of scattered light.

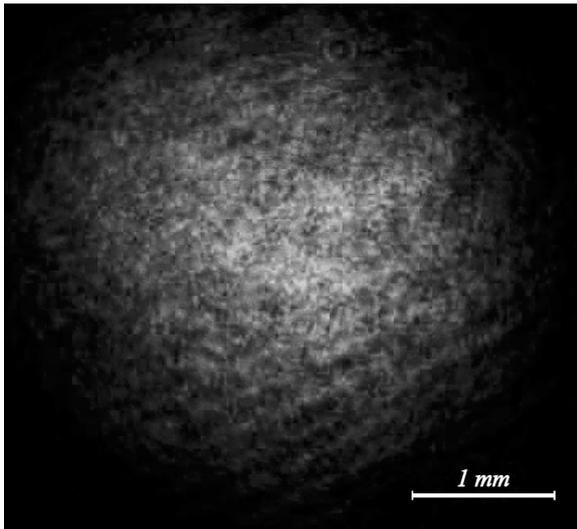


Fig. 2. The transverse image of the laser beam

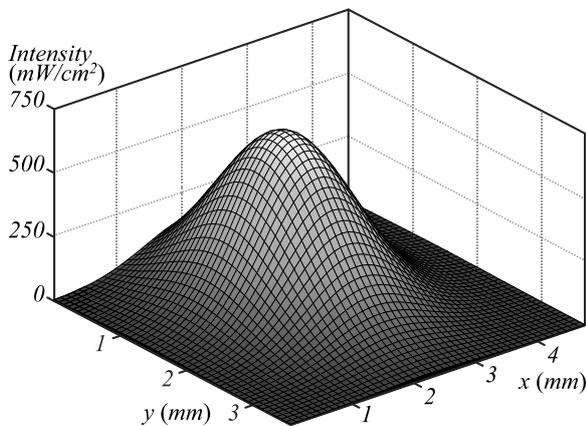


Fig. 3. The approximation of the Gaussian beam in the transverse direction

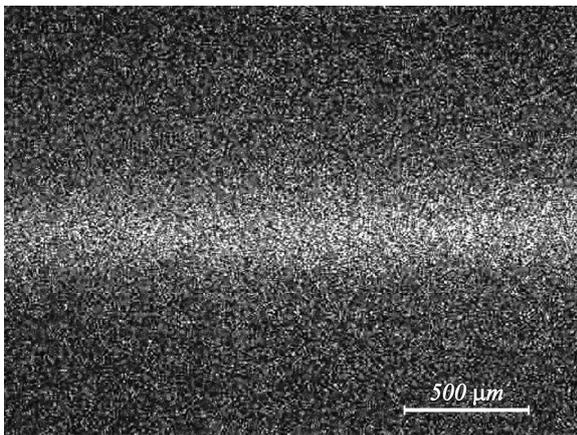


Fig. 4. Longitudinal image of the focal waist obtained by scattering the z-polarization of the Gaussian beam

A two-dimensional function approximated the image to determine the centerline of the beam in the longitudinal direction:

$$f(y, z) = a(z) \cdot \exp\left(-\frac{(y - y_0 - kz)^2}{2\sigma^2(z)}\right) + f_0(y, z), \quad (2)$$

$$a(z) = a_0 + a_1z + a_2z^2, \quad \sigma(z) = \sigma_0 + k_{\sigma 1}z + k_{\sigma 2}z^2,$$

$$f_0(y, z) = f_1 + k_zz + k_yy.$$

The intensity maximum change along the axis Oz (parameters  $a_0, a_1, a_2$ ), width change (parameters  $\sigma_0, k_{\sigma 1}, k_{\sigma 2}$ ), beam tilt, and position (parameters  $y_0$  и  $k$ ) were taken into account in the approximation function. The speckled inhomogeneous image background was also taken into account (parameters  $f_0, k_z, k_y$ ). The variation was made on these 11 parameters. The image approximation of the longitudinal intensity distribution of the beam waist by Eq. 2 is shown in fig. 5.

The bright background in the image (fig. 4) is due to the re-scattering of the radiation. That is why the contrast of the waist image is low. Nevertheless, the image approximation by Eq. 2 allowed us to determine the position of the centerline, the width, and the tilt of the beam quite precisely. In our experiments, the error of the transversal position of the centerline was  $\sim 0.1 \mu\text{m}$ .

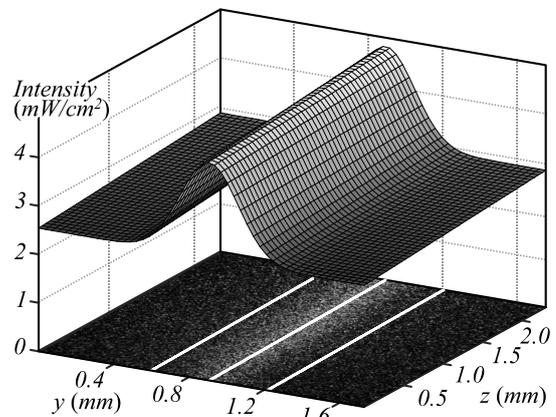


Fig. 5. The approximation of the z-polarization of the Gaussian beam waist image

## 2. Method of determination of the Bessel beam waist transversal position

To determine the transverse position of different electric field components (z- and y-polarization) of the asymmetrically converging Bessel beam (beam with non-zero intrinsic OAM) in the waist region, we used the experimental setup shown in fig. 6. To create a beam with non-zero intrinsic OAM, an enlarged Gaussian beam was passed through the diffractive optical element (DOE). The topological charge  $l$  of that beam was equal 2. Such a beam with a spiral wavefront we conditionally called the Bessel beam. On passing the beam through DOE, the intensity distribution in the transverse direction is shown in fig. 7. The CCD camera was fixed after the phase plate  $\lambda/4$ . The image was approximated by the function:

$$f(x, y) = a \cdot J_l^2\left(\frac{\sqrt{x^2 + y^2}}{R}\right) + f_0, \quad (3)$$

where  $J_l(r)$  is the  $l$ -order first kind Bessel function.

This function takes zero value in  $r = 5.14$  for  $l = 2$ . Therefore, the radius of the first dark ring equals  $r_1 = 5.14R$ . Furthermore, the beam passed through the

diaphragm D. Its radius was chosen to cut the central ring only.

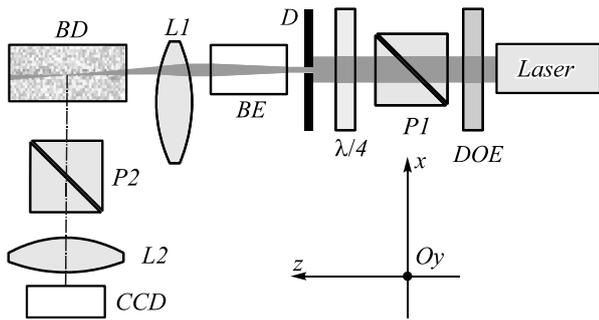


Fig. 6. The experimental setup for observation of the focal waist position of the beam with nonzero OAM. DOE is diffractive optical element; P1, P2 are polarizers; D is diaphragm; L1, L2 are lenses; BE is beam expander; BD is beam diffuser

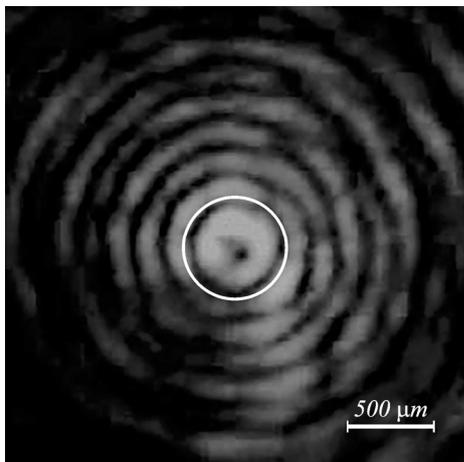


Fig. 7. The image of the Bessel beam in the transverse direction

An approximation of the transverse image of this part of the beam using Eq. 3 is shown in fig. 8.

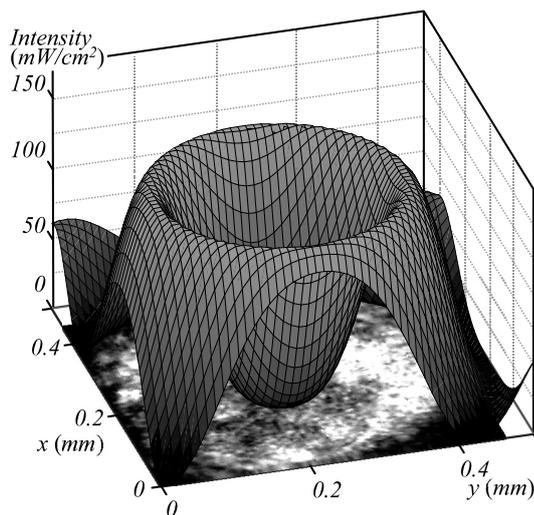


Fig. 8. The approximation of the Bessel beam in the transverse direction

The Bessel beam's central ring passed through the beam expander (BE). The BE enlarged the ring diameter by 12 times. The enlarged beam passed through the upper half of lens L1 to form an asymmetrically converged beam.

The system for registration of the scattered light was the same as in the experiment with Gaussian beam (fig. 1).

The longitudinal image of the y-polarization of the Bessel beam is shown in fig. 9. This image shows the intensity decrease in the middle of the waist in the longitudinal direction. Therefore, approximating this image by a Gaussian function will be incorrect. The intensity distribution in the beam waist was approximated by the function (4):

$$f(y, z) = a(z) \cdot F\left(\frac{y - y_0 - kz}{s}\right) + f_0(y, z) \quad (4)$$

$$a(z) = a_0 + a_1z + a_2z^2,$$

$$f_0(y, z) = f_1 + k_zz + k_yy + k_{z2}z^2 + k_{y2}y^2 + k_{yz}yz,$$

$$F(y) = \int_{-\xi}^{+\xi} J_l^2(\sqrt{x^2 + y^2}) dx, \quad -r \leq y \leq r,$$

$$\xi = \sqrt{r^2 - y^2}.$$

The primary function is  $F(y)$ , in which the Bessel intensity distribution in the beam cross-section is taken into account. This function characterizes the scattered Bessel beam's projection onto the  $Oyz$  plane, in which the CCD camera was placed. Secondary scattering in the medium was not taken into account,  $y_0$  is the position of the central line of the waist in the transverse direction, and  $s$  is its width. The coefficient  $k$  depends on the tilt of the waist image. The difference in intensity along the waist ( $a_0, a_1, a_2$ ), and the background irregularity ( $f_1, k_z, k_y, k_{z2}, k_{y2}, k_{yz}$ ) were taken into account in Eq. 4 also. Equation 4 has 12 parameters, by which the variation was made. The result of image approximation (fig. 9) by this function is shown in fig. 10.

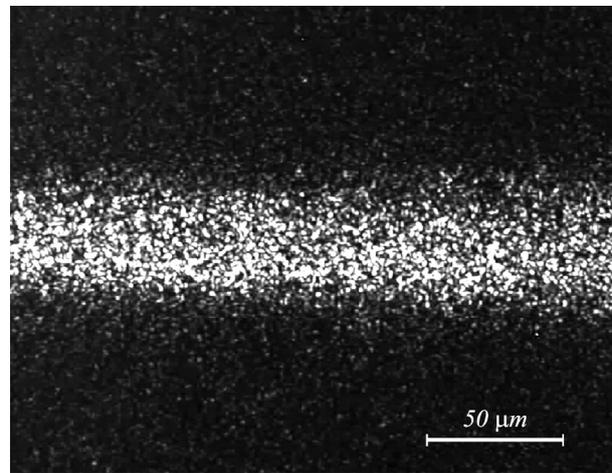


Fig. 9. The image of the y-polarization of the Bessel beam waist

The most impressive is the image processing the z-polarization of the Bessel beam waist. According to the theoretical predictions, namely, this component image could demonstrate the effect of spin-orbital interaction. However, a difficulty arises from the low intensity and low contrast of this image (fig. 11). Nevertheless, the approximation of this image by Eq. (4) allows us to determine the centerline of the Bessel's beam waist in the transverse di-

rection (along the axis  $Oy$ ) with accuracy  $\sim 0.1 \mu\text{m}$ . Besides, such an approximation allows us to determine its width and tilt of Bessel beam waist (fig. 12).

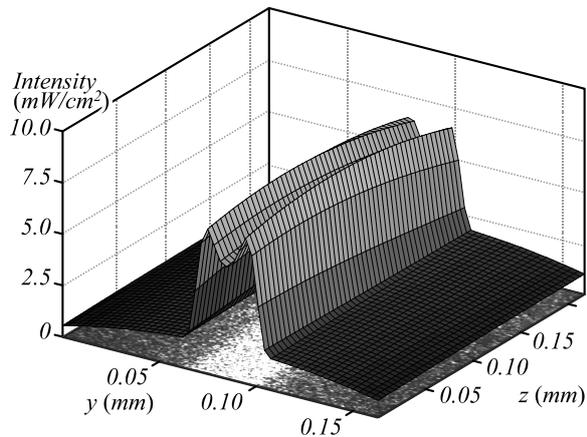


Fig. 10. The approximation of the  $y$ -polarization of the Bessel beam waist

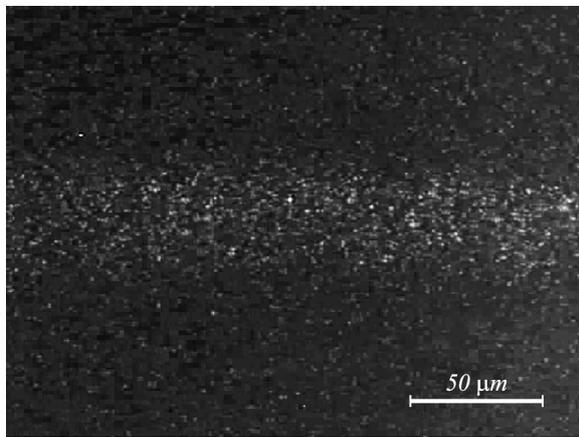


Fig. 11. The image of the  $z$ -polarization of the Bessel beam waist

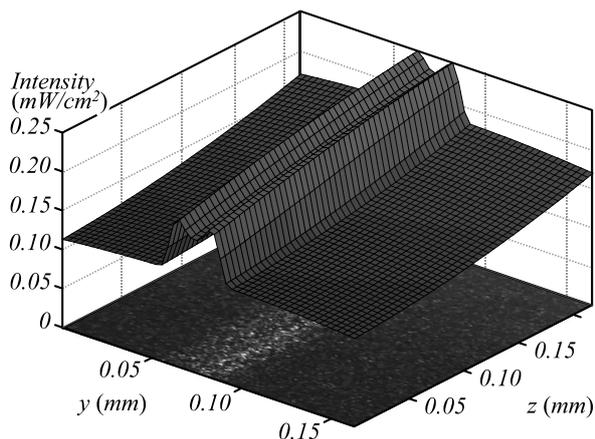


Fig. 12. The approximation of the  $z$ -polarization of the Bessel beam waist

We have demonstrated the effectiveness of the developed method for determining the shift of the waist of an asymmetric converging Gaussian beam under the circular polarization sign change. Fig. 13 shows the intensity distribution of the  $z$  components of the left and right circular polarized Gaussian beam in the waist region. The devel-

oped method has allowed us to detect the waist shift by the value  $0.4 \pm 0.1 \mu\text{m}$ . We should emphasize that the image's shift was recorded visually at the first experimental observation of the effect. The waist position was determined by the lines drawn in the photographs [13, 36]; the shift value was approximately  $1.5 \mu\text{m}$ . Thus, the developed method will allow us to study the waist shift effect for different light beams in detail.

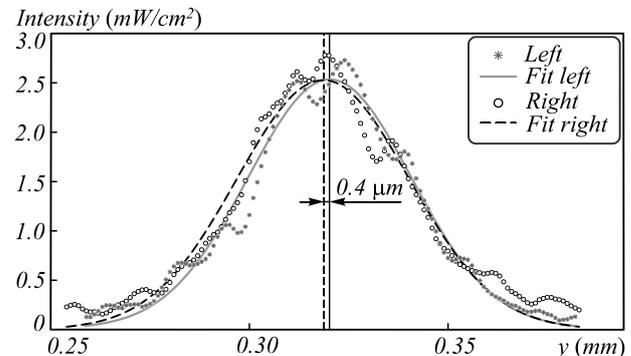


Fig. 13. The intensity distribution of the  $z$ -polarization in the beam waist and approximation by a Gaussian function for left and right circular polarization

### Conclusion

We have proposed approximating noisy images of low intensity and low contrast without clear boundaries by special functions. The method turned out to be very effective for determining the beam's waist main characteristics (centerline position, width, tilt). We were successful in determining the beam waist longitudinal component's position with an accuracy of  $0.1 \mu\text{m}$ . The developed method can be useful for studying the fine effects of the spin-orbit interaction of light in beams with different phase and intensity structures.

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