Numerical approach to compound quantum repeater scheme with coherent states

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Abstract

A numerical model of a quantum repeater operating with Schrödinger cat states is constructed. The model describes the performance of such a system in the presence of decoherence effects, namely, noise in the quantum channel and the efficiency of the photon-number-resolving detector. In the framework of the numerical model, a theoretical analysis of the system functioning is carried out for the elementary link by calculating its performance characteristics. Namely, we calculate photodetector click probabilities and fidelity for various sets of decoherence parameters. These estimates are necessary in the context of further experimental research at the junction with other branches of quantum communications, so that to use the entanglement distribution when it comes to operating quantum teleportation and quantum key distribution protocols based on entanglement. The model will be developed further as a versatile drag-and-drop software simulating the full-fledged entanglement swapping protocol operation.

Keywords: quantum repeater, entanglement swapping, decoherence, quantum key distribution.

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Introduction

The distribution of quantum states over long distances carries enormous importance for many practical applications in quantum technologies, such as quantum key distribution (QKD) and quantum networks. In practice, quantum channels, be it optical fibers or free space, impair losses due to decoherence, which limits the characteristic distances over which information can be transmitted directly by sending individual quantum objects. In classical telecommunications, this urgent problem of optical losses is solved by optical amplifier implementation. However, such an approach is not applicable to the case of quantum communications, for there is the no-cloning theorem. Nonetheless, the problem can be solved with a rather sophisticated entanglement-based technique known as the "quantum repeater" [1, 2]. The idea behind it suggests that the phenomenon of quantum entanglement be employed, namely, entanglement distribution and entanglement swapping procedures, as well as quantum teleportation.

Much attention is paid to the theoretical study and experimental implementation of quantum repeaters. By 2023, theoretical models of quantum repeater systems based on various physical principles (for example, based on trapped ions, or even completely photonic systems) have already been proposed [3]. Particularly, analytical models were developed to describe the phenomenon of decoherence for Schrödinger cat states in the context of quantum repeater realization. One of the approaches is to solve Lindblad master equation describing the nonunitary dynamics of a system density matrix, whereas the second one considers a beamsplitter concept, where the medium acts as an additional mode, and the energy leakage is described by the beamsplitter operation. It should be noted that the performance of quantum repeaters allowing for decoherence effects has not been described by numerical methods yet.

Thus, in this work, we firstly propose a solution to the problem of numerical analysis and evaluation of quantum repeater performance in Lindblad dynamics, accounting for a noisy fiber quantum channel. Previously, the phenomenon of decoherence in a channel was considered analytically in the context of thermal Markov baths for Schrödinger cat states [4].

A comprehensive analysis of a quantum repeater elementary link performance is carried out, accounting for the non-ideal components of the system in order to use the obtained results and dependencies for the development of practical implementations of the system. The analysis is performed in terms of such system performance characteristics as photodetector click probability and fidelity. The constructed numerical model opens up a wider functionality for considering the dependencies of system characteristics on its parameters so that to evaluate its performance for further experimental realizations. The model is a set of separate blocks (i.e., scheme elements), each described by its own corresponding physical model. Such a scheme should be regarded as the basis for a quantum network simulator consisting of many elementary nodes and allowing the entanglement swapping protocol implementation. Next, the realization of such a simulator in the format of an element-by-element drag-and-drop product will be presented.

1. Quantum repeater model

1.1. Schrödinger cat states

The so-called Schrödinger cat states (even and odd) [5] are either a symmetric (for even cat-states) or antisymmetric (for odd cat-states) superposition of coherent states. Their mathematical representation is as follows:

$$|\alpha_{\pm}\rangle = \frac{1}{\sqrt{M_{\pm}(\alpha)}}\left(|\alpha\rangle \pm |-\alpha\rangle\right), \tag{1}$$

where $|\pm\alpha\rangle$ stands for a coherent state of an amplitude α and $M_{\pm}(\alpha) = 2(1 \pm \exp(-2|\alpha|^2))$ is the normalization factor.

The corresponding Wigner function distribution of such a state (for an even superposition case) takes the following form:

$$W(\beta) = \frac{M_{+}^{2}}{\pi}\exp\left(-2|\beta-\alpha|^2\right) + \exp\left(-2|\beta+\alpha|^2\right) + \\ +2\exp\left(-2|\beta|^2\right)\cdot\cos\left(Im(\beta)\right). \tag{2}$$

The resulting distribution depicted in Fig. 1(*a*) clearly illustrates the mathematical definition: the two Gaussian peaks of the individual coherent states feature oscillations resulting from their quantum interference in between. After passing through a beamsplitter (Fig. 1(*b*)), i.e. already impairing decoherence, such a state loses its central interference oscillations, thus becoming a mixed state with closer peaks of a higher amplitude.

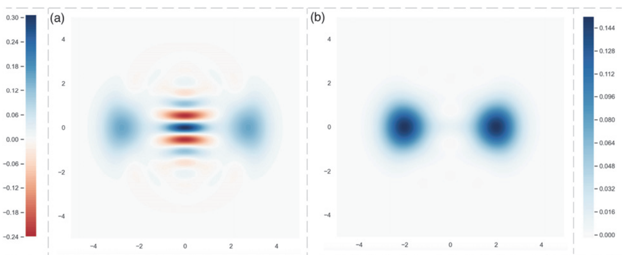

Fig. 1. Wigner function of the (a) even cat-state of an amplitude $|\alpha|^2 = 4$ and (b) the same state after passing through a beamsplitter

Noteworthy, the spectrum for Schrödinger cat states' applications is quite wide, starting from metrology [6, 7], where even cat-states in combination with the parity method are used to implement super-resolution measurements of angular rotation, to quantum computing [8], teleportation [9], and QKD [10]. But here one should place emphasis on the fact Schrödinger cat states feature great potential for entanglement creation needed to establish a quantum repeater.

1.2. Elementary link of a quantum repeater

The simplest case considered, a quantum repeater comprises a sequence of elementary links employed to create entanglement between pairs of states [2]. The scheme of the latter is presented in Fig. 2.

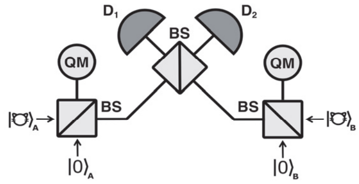

Fig. 2. Schematic representation of a quantum repeater elementary link

Here, two quantum memory (QM) cells are utilized, as well as photon-number-resolving photodetectors (D_1 and D_2) and beamsplitters (BS). First, Schrödinger cat states are generated on both parties' sides. Next, the states are passing through the beamsplitters, one part stored in a memory cell, and another transmitted through a quantum channel to be combined at a 50/50 beamsplitter and, finally, detected. The detection of a single photon heralds the storage of an entangled coherent state in a QM cell.

Thus, additional misinterpretation might arise from the detectors themselves, for they are characterized by their efficiency ξ, which does not equal to unity, realistic conditions considered.

2. Performance characteristics and approaches to their mathematical description

To qualitatively estimate the negative impact of decoherence on the performance of a quantum repeater elementary link, we here address two characteristics mainly, that are photodetector click probability (the probability of photocounts) and the fidelity between input and output states.

2.1. Probability of photocounts

Let us address the mathematical formalism of a photon-number-resolving photodetector first. The action of such photodetectors is characterized by the positive-operator-valued measure (POVM) of a form [11]:

$$\hat{\Pi}_k =: \frac{(\xi\hat{n})^k}{k!} e^{\xi\hat{n}} :, \tag{3}$$

$$\hat{n} = \hat{a}^{\dagger}\hat{a}, \tag{4}$$

where $\hat{a}\left(\hat{a}^{\dagger}\right)$ is the bosonic annihilation (creation) operator, ξ denotes the detector efficiency, and the notation :…: corresponds to the normal order of operators.

The resulting conditional probability to detect m photons in the given state is then estimated using the well-known Kelley–Kleiner formula [11, 12]:

$$P_{\xi}(k)=\mathrm{tr}\left[\rho:\frac{\left(\xi\hat{a}^{\dagger}\hat{a}\right)^{k}}{k!}e^{\xi\hat{a}^{\dagger}\hat{a}}:\right]=\sum_{n=k}^{\infty}\rho_{nn}\binom{n}{k}\xi^{k}\left(1-\xi\right)^{n-k}, \quad (5)$$

where ρ is the measured state density matrix representation with $\rho_{nn}\equiv\langle n|\rho|n\rangle=p_{\xi=1}(n)$.

2.2. Fidelity

A well-known way to estimate the accuracy of the "similarity" between two quantum states is the fidelity quantity. For the initial quantum state ρ and the resulting one σ it reads:

$$\mathcal{F}(\rho,\sigma)=\left(\mathrm{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^{2}. \quad (6)$$

So, maximum fidelity value $\mathcal{F}(\rho,\sigma)=1$ corresponds to an ideal state reproduction.

3. Decoherence consideration

3.1. Dissipative evolution in a noisy quantum channel: Lindblad master equation

In the context of the article, the analysis follows a general approach to describing a noisy quantum channel through the representation of its dynamics in the form of Lindblad master equation for the density matrix of the system. The equation describes the system's nonunitary (dissipative) evolution, which is represented by a completely positive trace-preserving map [13]:

$$\dot{\rho}(t)=-i[H,\rho]+\sum_{i}\left(L_{i}\rho L_{i}^{\dagger}-\frac{1}{2}\left\{L_{i}^{\dagger}L_{i},\rho(t)\right\}\right)\equiv\mathcal{L}\rho(t), \quad (7)$$

where ρ is the system density matrix, H is the interaction Hamiltonian, and L_i are the so-called jump-operators that describe the very dissipative dynamics itself.

In the work, the Lindblad equation is solved numerically in the QuTip quantum Toolbox in Python [14].

3.2. Efficiency of a photon-number-resolving photodetector

Obviously, the efficiency parameter ξ of a photon-number-resolving photodetector influences the detection results significantly.

To demonstrate the way it does so, let us have a look at Fig. 3. The numerical simulation results show the photocount probability for a photon-number-resolving detector featuring three various efficiencies for the case of (a) even and (b) odd cat-states ($|\alpha|^2=4$).

From the dependencies obtained, it is evident that the probability of odd (even) number of clicks vanishes for the symmetric (antisymmetric) states, an ideal detector being the case, thus, suggesting perfect distinguishability of the cat states. The tendency is though that the pattern impairs blurring and becomes more of a saw-toothed shape as the detector efficiency degrades. It, in turn, indicates it becomes a more complicated task to subtract accurate results at the output of an elementary link even without channel losses introduced.

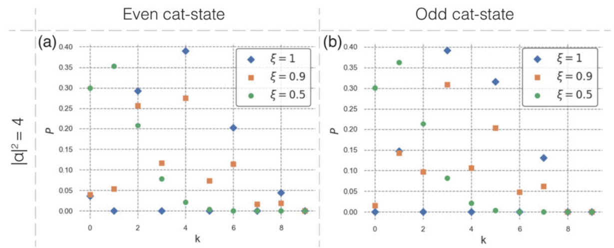

Fig. 3. The probability of photocounts for an (a) even cat-state and (b) odd cat-state of an amplitude $|\alpha|^2 = 4$ for various detector efficiency ξ values

4. Results and discussion

After the approaches and principles employed were discussed, let us now switch to the very performance analysis.

First, a look was taken on the way the probability of photocounts behaves depending on an input cat-state amplitude. Since dealing with a quantum repeater work the aim is for one to distinguish between even and odd Schrödinger cat states, the probabilities were addressed correspondingly: for each state, the probability was considered as an overall sum of all even and odd number of clicks for an initial state. The probability of zero clicks, i.e. vacuum state detection, was calculated and plotted separately. For further clarity, it should be highlighted that, in the context of work, the initial even cat-state does contain the vacuum component in itself, while the detection probabilities for output states, as mentioned, are calculated in terms of vacuum and even number of clicks (for $k=2$ and above) separately. The reason for such a discrepancy in the notation is that the zero component lacks any informative content, especially for the subsequent entanglement swapping procedure. Such delimitation allows one to discern ranges of optimal state discrimination in terms of vacuum separation.

Analytically, there is a possibility to remove the zero component from an even Schrödinger cat state and take a look at the pure photocount probability dependencies [15]. However, one must bear in mind that it is yet a mathematical trick that does not resolve the challenge experimentally.

Back to the main point now, the results obtained are presented in Fig. 4.

It can be observed that the behavior of the graphs features certain similarities. Namely, the smaller is the amplitude of an input state, the less is the probability to successfully distinguish between the states, even cat-state as an input featuring higher values. Moreover, increasing the noise in a quantum channel, one naturally observes the decrease in the corresponding useful information about the output state, getting a high vacuum detection probability for the whole amplitude range.

Next, similar dependencies were obtained for the case of various channel noise coefficients η for even and odd cat-states of two amplitudes ($|\alpha|^2=1$ and $|\alpha|^2=4$). The graphs are given in Fig. 5.

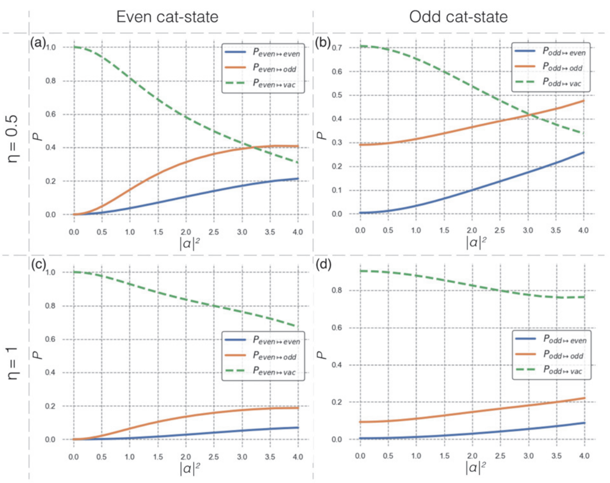

Fig. 4. The dependence of photocount probabilities on the initial state amplitude for the cases of even (a, c) and odd (b, c) cat-states for two channel loss coefficients

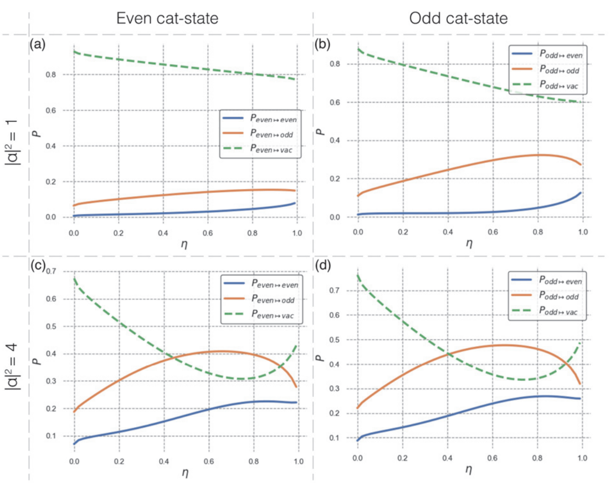

Fig. 5. The dependence of photocount probabilities on the channel loss coefficient for the cases of even (a, c) and odd (b, d) cat states for two initial cat-state amplitudes

One can notice several peculiarities in the resulting curves. First, for an even cat state, the probability to detect an even number of clicks is less than to detect an odd amount of photons for all the cases considered. Additionally, for bigger cat-states ($|\alpha|^2 = 4$), there is a range of a well-pronounced vacuum state detection minimum, thus the distinguisibility is better there.

Finally, fidelity curves' behavior was analyzed in terms of both $|\alpha|^2$ and η (see Figure 6). The former were obtained for $\eta = 1$, while the latter – for $|\alpha|^2 = 4$.

Fixed amplitude being the case, the curves are qualitatively near-identical, showing an obvious deterioration of fidelity as the channel losses increase. The situation becomes different when it comes to fidelity dependence on the input state amplitude. Here, one observes its steady decrease with an even cat-state amplitude increase (for both lossless and noisy channels). Regarding odd cat-states, however, if a quantum channel does not introduce additional dissipation, fidelity, first, preserves its value of around $\mathcal{F} = 0.5$, and then starts to increase at a certain point ($|\alpha|^2 = 3$ and above). Meanwhile, a decrease in fidelity is observed for a noisy channel.

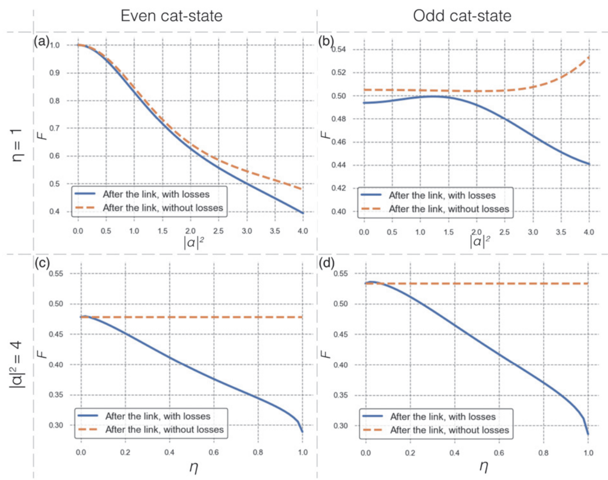

Fig. 6. The dependence of the fidelity value (a, b) on the initial state amplitude for a fixed channel loss coefficient (h = 1) and (c, d) on the channel loss coefficient for a fixed input state amplitude ($|\alpha|^2 = 4$)

Conclusion

In this work, a universal numerical model of a quantum repeater operating with Schrödinger cat states was constructed, describing the performance of such a system in the presence of decoherence effects (noise in a quantum channel and realistic (i.e., $\xi \neq 1$) photon-number-resolving detector). The numerical model was used to analyze a quantum repeater elementary link performance by calculating photodetector click probabilities and fidelities for various sets of decoherence parameters. Contributions of the decoherence effects on the performance characteristics of a potential quantum repeater were addressed separately and in combination. Conclusions regarding even and odd Schrödinger cat states were made in connection with their implementation in a quantum repeater elementary link. The analysis is of a high use for experimental research and practical realizations of quantum repeaters. Next, the presented model will serve as a part for a versatile drag-and-drop entanglement swapping protocol operation simulator.

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