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ON THE DESIGN OF LASER RADIATION FOCUSERS WITHIN THE DIFFRACTION APPROXIMATION

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Abstract—A technique is proposed for designing optical elements focusing radiation into a volume domain in space with specified intensity distribution in the domain. A focuser has been designed giving an almost rectangular intensity distribution from a beam with gaussian intensity profile.

The development of laser technology generated a broad class of problems, related to the transformation of optical radiation, which could not be solved by using conventional optical elements. One of these is the focusing of radiation into a given spatial region with specified intensity distribution. For this purpose one uses specially designed and manufactured phase plates, the focusers [1]. The methods that have been developed [2] in focuser design rely on the geometrical optics approximation and are oriented towards focusing the radiation into an arbitrary curve. However, if the dimensions of the focusing region are small, diffraction effects must be taken into account. Some design methods rely on solving a nonlinear integral equation [3] that relates the intensity distribution in the focal plane to the phase profile of the focuser, but an exact solution of this integral equation may not exist for certain focusing domains. This situation is typical for three-dimensional (volume) domains.

The present work proposes a focuser design method which accounts for diffraction. It is based on an iterative scanning procedure to locate the focuser phase profile which will produce an intensity distribution very close to the specified one. Let the complex amplitude $A(\vec{r}, z = 0)$ characterize the radiation at the exit plane $(z = 0)$ of the focuser:

$$A(\vec{r}, 0) = A_0(\vec{r}) \exp[iu(\vec{r})], \tag{1}$$

where $u(\vec{r})$ is the phase of the field, $A_0(\vec{r})$ is its amplitude, assumed known, and $\vec{r}\{x, y\}$ are cartesian coordinates in a plane perpendicular to the OZ-axis.

We shall describe the propagation of radiation to the focusing region $L \leqslant z \leqslant L + l$, separated by segments of optically nonuniform media, in terms of the quasioptical approximation:

$$2ik\frac{\partial A}{\partial z} = \Delta_{\perp} A + \frac{2k^2}{n_0}\,\delta n A, \quad \Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{2}$$

Here $k = 2\pi/\lambda$, λ is the wavelength of the radiation, δn and n_0 are the fluctuation and the unperturbed refractive index of the medium, and L is the distance between the focuser and the focusing region of length l. For a specified intensity distribution $I^0(\vec{r})$ the focusing quality in a given region may be estimated by an integral criterion, for example,

$$J = \int\!\!\int |AA^* - I^0|_{|z=L}\, d^2\vec{r}. \tag{3}$$

(Here and below we shall consider, for simplicity, only a plane focusing domain. Generalization to a volume domain is not complicated, one just has to add a further integration over z from L to l in the criterion J.)

One must select the focuser phase $u(\vec{r})$ so as to minimize the functional J and thereby form that intensity distribution in the focusing domain which is closest to the required one. This formulation of the problem has a series of important advantages. In particular, it allows for an approximate solution to the focusing problem whenever the exact solution $AA^* = I^0$ does not exist. It also enables one to solve the problem of focusing into a specified domain through nonuniform and nonlinear media with given refractive index nonuniformities.

The search for the minimum of J is best performed by a gradient method [4] whereby the minimizing set $\{u_n\}$ may be constructed in terms of a known gradient J' in the following manner:

$$u_{n+1}(\vec{r}) = u_n(\vec{r}) - h_n \cdot J'\{u_n\}, \tag{4}$$

where h_n is the step length in the iteration.

By analysing the change ΔJ caused by a small variation u, it can be shown that J' is given by

$$J'\{u\} = -2A_0[|\psi| \sin(u + \arg \psi)]_{|z=0}, \tag{5}$$

where the "adjoint" function ψ satisfies the conditions

$$-2ik\frac{\partial \psi}{\partial z} = \Delta_\perp \psi + \frac{2k^2}{n_0}\delta n \psi, \quad 0 \leqslant z \leqslant L; \tag{6}$$

$$\psi(\vec{r}, z = L) = [A^* \operatorname{sign}(AA^* - I^0)]_{|z=L}. \tag{7}$$

We therefore have the following scheme for determining the optimal focuser phase profile $u(r)$:

(1) choose an initial phase function $u_0(\vec{r})$;
(2) solve the problem of radiation propagation from the focuser aperture ($z = 0$) to the focusing region ($z = L$), in terms of Eqs (1) and (2);
(3) find the complex amplitude of the "adjoint" function $\psi(\vec{r}, L)$ by using the boundary condition (7);
(4) in accordance with (6) solve the inverse propagation problem for ψ and determine $\psi(\vec{r}, 0)$;
(5) choose a new value for the focuser phase profile, $u_1(\vec{r})$, for example by using Eqs (4) and (5), and reiterate from step (2).

The propagation equations (2) and (6) are best solved by decoupling into physical factors [5] using fast Fourier transform algorithms, which makes for a comparatively low numerical expenditure invested into the design scheme.

As an example, the design problem was solved numerically for the almost straight line intensity distribution $I^0(\vec{r}, z) \sim \exp[-(x/b_0)^{10} - (y/b_0)^{10}]$ in the domain $L \leqslant z \leqslant L + l$, from an initial gaussian beam $A_0 \sim \exp(-r^2/b^2)$. The propagation medium was assumed linear and optically

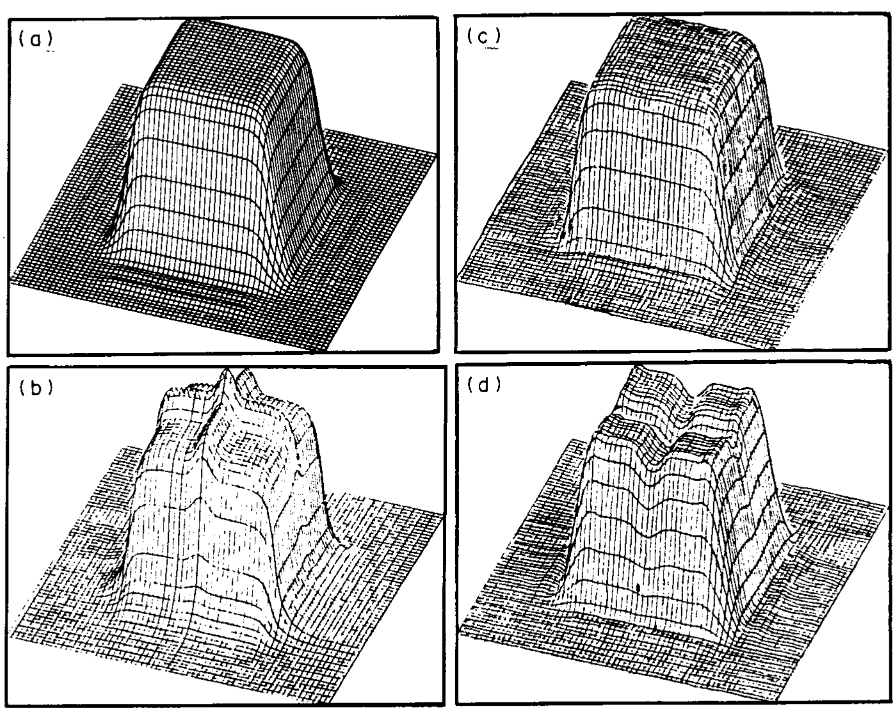

Fig. 1. Focusing into a volume region ($0.10L_g \leqslant z \leqslant 0.14L_g$): (a) the specified intensity profile $I^0(\vec{r})$; the intensity distribution $|A(\vec{r}, z_k)|^2$ generated by the focuser at the planes: (b) $z_1 = 0.10L_g$, (c) $z_2 = 0.12L_g$, (d) $z_3 = 0.14L_g$.

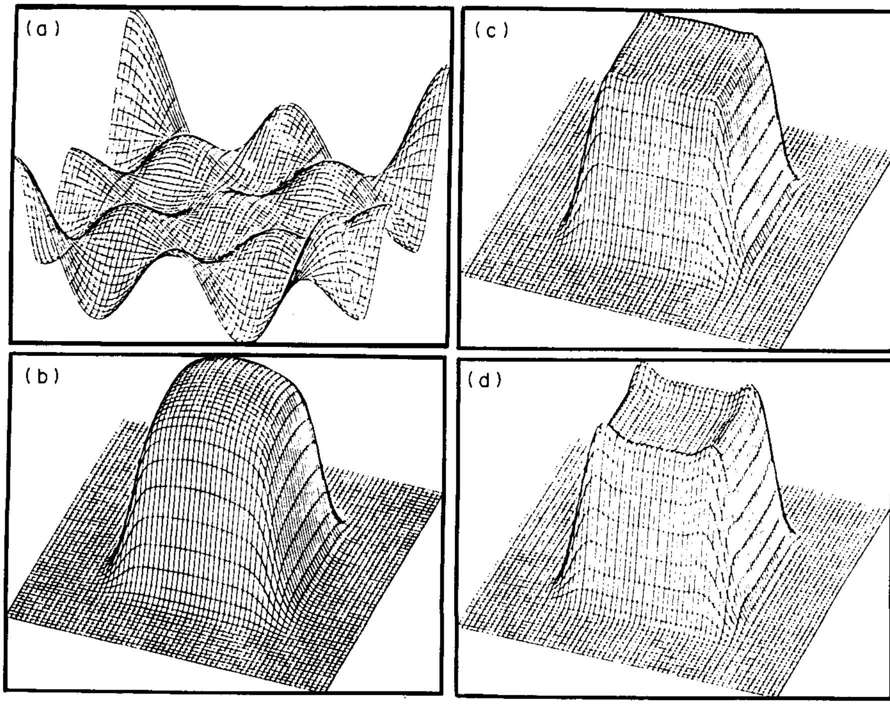

Fig. 2. Focusing into a volume region ($0.10L_g \leqslant z \leqslant 0.14L_g$) by flexible mirror ($M = 4$): (a) mirror surface profile; the intensity distribution $|A(\vec{r}, z_k)|^2$ formed at z_k: (b) $z_1 = 0.10L_g$, (c) $z_2 = 0.12L^g$, (d) $z_3 = 0.14L_g$.

uniform. The specified intensity distribution $I^0(\vec{r})$ is illustrated in Fig. 1a, while Figs 1b,c,d show the intensity profiles in the planes $z_k = L + (1/2)(k - 1)$, $k = 1, 2, 3$, obtained by minimizing the criteria

$$J_\Sigma = \sum_{k=1}^{3} \int\int |AA^* - I^0|_{|z=z_k} \, d^2\vec{r}$$

for moderate diffraction ($L = 0.1\ L_g$, $l = 0.04\ L_g$, $L_g = kb_0^2/2$ is the diffraction length for a given focuser aperture width b_0). We note that the width of the diffraction-limited spot at $z = L$ is $b_g = b_0 L/L_g = 0.1 b_0$, ten times smaller than the width of the specified intensity distribution $I^0(\vec{r})$. It can be seen from Fig. 1 that under the given conditions one gets quite close to the specified intensity distribution in the whole focusing region.

A promising idea is to use flexible controllable mirrors as focusers, so that the phase field, after reflection from these, may be represented in the form $u(\vec{r}) = \sum_{m=1}^{\mu} a_m S_m(r)$, where a_m is the control signal, $S_m(\vec{r})$ is the mirror response function, and M is the number of driven mirrors. The functional J_Σ thereby becomes a function of the control signals a_m: $J_\Sigma\{a_m\}$. The previous focusing problem in the region $L \leqslant z \leqslant L + l$ ($L = 0.1\ L_g$, $l = 0.04\ L_g$) was solved for a mirror whose response function was modelled by the Hermite function $S_m(x) = H_{2m}(x/b_0)\exp(-x^2/2b_0^2)$, ($M = 4$), where $H_{2m}(x)$ is the Hermite polynomial. Figures 2b,c,d show the field intensities in the planes z_k, $k = 1, 2, 3$ for optimal focusing (ensuring $\min_{\{a\}} J_\Sigma\{a_m\}$). The surface profile of such a flexible mirror is shown in Fig. 2a. Thus by using flexible mirrors whose surface profile stays fairly smooth one can achieve qualitatively the required intensity distribution.

We note that diffraction limits the extent of the focusing region in which it still makes sense to discuss the problem of focusing a given intensity distribution. Thus, already for $L = 0.5L_g$ (the focusing region being only twice as large as b_g) the intensity distribution obtained AA^* differs significantly from the specified $I^0(\vec{r})$, although it still has a pronounced central plateau followed by a smooth drop.

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