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THREE-DIMENSIONAL SYMMETRIC BIFOCAL SYSTEMS

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Abstract—A method for designing three-dimensional bifocal symmetrical systems is discussed. Parameters characterizing these systems are introduced. Consideration is given to the symmetry of the initial wavefront and to the dependence of the solution on the initial parameters. Smooth surfaces have been obtained for both mirrors, and conditions derived under which the larger mirror may be a surface of revolution.

Nonspherical mirror systems enable one to obtain effects normally unattainable with optics based on spherical surfaces. Aspherical surfaces, whose precision is dictated by the wavelengths in use, have so far been applied to comparatively long wavelengths (submillimetre to centimetre). Reproducible precision of these surfaces is achieved by using numerically controlled machines in their manufacture.

One of the promising recent developments is the bifocal system [1], in which aberrations are completely eliminated in two plane-wave directions. For a given number of reflecting (or refracting) surfaces, bifocal systems have fewer aberrations than previously known systems. In addition, their angular resolution is 3 to 5 times that available in Cassegrain and Schwarzschild type reflecting systems [2]. Although all bifocal systems are three dimensional, the original theory was formulated in two-dimensional terms [3, 4].

The solution of the two-dimensional problem was presented in [4], which then went on to treat the resulting profiles as forming axisymmetric mirrors. The solution obviously possesses aberrations, even for two specified directions of nonaberrational focusing. A more consistent design scheme was presented for three-dimensional bifocal systems in [5], which carried out an extension from two- to three-dimensional systems. Below we set forth a different method of designing a three-dimensional bifocal system. The advantage of our algorithm is that it allows for analysing the dependence of the solution on the initial parameters, so that by suitably varying the latter, desired system parameters may be achieved (e.g. one of the mirrors may be constructed as a surface of revolution).

The goal of this paper is to design two reflecting surfaces which transform two spherical waves emerging from two focal points into two plane waves. The design of reflecting surfaces is carried out within the geometrical optics approximation, using the laws of specular reflection and the constancy of optical paths (law of Malus). Given the position of the focal point $\vec{r}_F$, the direction of the plane wave $\vec{n}$, and the optical path length S from the focus to the plane front, there is a one-to-two correspondence between the relevant points of both mirrors and the direction-normals to the mirror surfaces at these points. Indeed, given a point $\vec{r}$ on one of the mirrors, and the normal to the surface there, v, the corresponding point, $\vec{R}$ and its normal $\vec{N}$ at the other mirror are given by

$$\vec{R} = \vec{r} + \vec{k}\{S - |\vec{r} - \vec{r}_F|(1 - \vec{n}\vec{p})\}/(1 - \vec{n}\vec{k}); \quad (1)$$

$$\vec{N} = (\vec{n} - \vec{k})/\{2(1 - \vec{n}\vec{k})\}^{1/2}, \quad (2)$$

where $\vec{p} = (\vec{r} - \vec{r}_F)/|\vec{r} - \vec{r}_F|$, $\vec{k} = \vec{p} - 2\vec{v}(\vec{v}\vec{p})$, and the position of the wave front is chosen so that it passes through the focal point. If we specify a point and its normal $\vec{R}$ and $\vec{N}$ on the other mirror (Fig. 2), then the corresponding point and its normal at the surface of the first mirror, $\vec{r}'$ and $\vec{v}'$, are determined by

$$\vec{r}' = \vec{R} + \vec{e}L; \quad (3)$$

$$\vec{v}' = (\vec{q} - \vec{e})/\{2(1 - \vec{q}\vec{e})\}^{1/2}, \quad (4)$$

where

$$\vec{e} = 2\vec{N}(\vec{N}\vec{n}') - \vec{n}'; \quad (5)$$

$$L = 0.5\{(S' + \vec{n}'(\vec{R} - \vec{r}'_F))^2 - (\vec{R} - \vec{r}'_F)^2\}/\{S' + (\vec{e} + \vec{n}')(\vec{R} - \vec{r}'_F)\}; \quad (6)$$

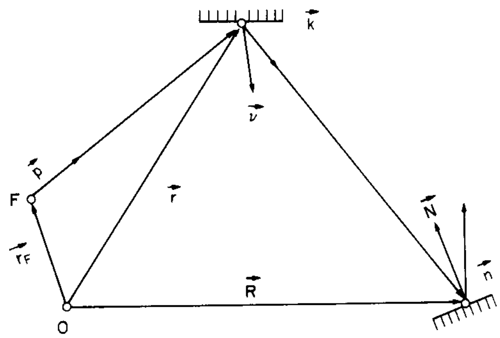

Fig. 1.

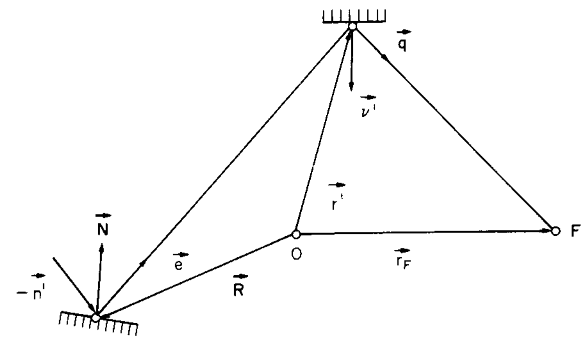

Fig. 2.

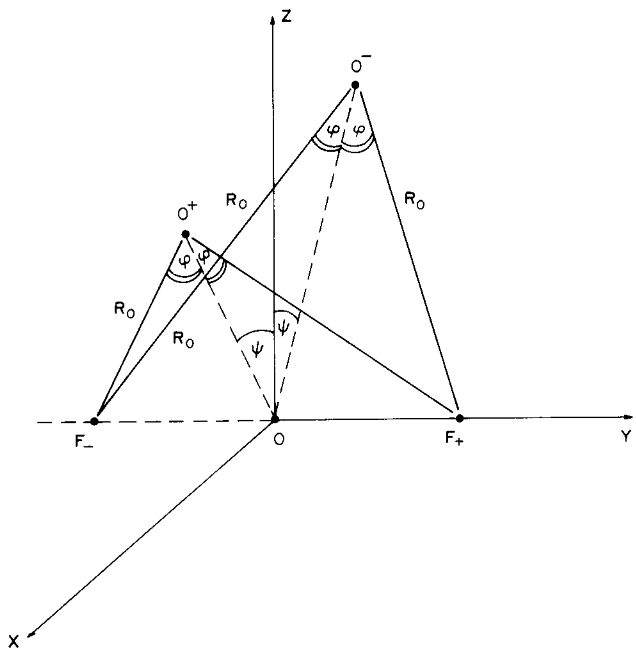

Fig. 3.

$$\vec{q} = -(\vec{e}L + \vec{R} - \vec{r}'_{\mathrm{F}})/\{S' - L + \vec{n}'(\vec{R} - \vec{r}'_{\mathrm{F}})\}\,. \tag{7}$$

If $\vec{n}$, $\vec{n}'$ and $\vec{r}_{\mathrm{F}}$, $\vec{r}'_{\mathrm{F}}$ are the same, then expressions (3)–(7) for $\vec{r}'$, $\vec{v}'$ reduce to formulae (1), (2) for $\vec{r}$ and $\vec{v}$. In the opposite case $\vec{r}'$, $\vec{v}'$ will differ from $\vec{r}$ and $\vec{v}$ for the same values of $\vec{R}$ and $\vec{N}$. We can then fix $\vec{R}'$ and $\vec{N}'$ corresponding to $\vec{r}'$ and $\vec{v}'$ by using (1) and (2). We are thereby led to the construction of bifocal systems, because in these the positions of the foci and the directions of the normal to the plane fronts are different. The algorithm is started by choosing an initial point $\vec{r}_0$ on one of the mirrors, and the corresponding normal to the mirror surfaces $\vec{v}_0$. By using the set of relations (1), (2) and (3)–(7) we then produce a certain set of so-called reference points. For an arbitrary assignment of the foci F and F', the normal to the wave front, the optical path lengths S and S', the initial point $\vec{r}_0$ and the corresponding normal $\vec{v}_0$, one obtains a nonsymmetrical system of reference points. In order to get a symmetrical system the initial condition must be chosen in a special way.

The system is taken to possess two planes of symmetry. We arrange two focal points symmetrically with respect to one of the planes, the unit normals to the plane front are symmetric with respect to the other plane, arranged in such a manner that the focal points and normals to the wave front lie in the same plane. A rectangular Cartesian system is introduced, as shown in Fig. 3. In this system the focal points are along the OY axis and the normals to the plane front have the components

$$\vec{n}_+ = \{0, -\sin\alpha, \cos\alpha\}; \qquad \vec{n}_- = \{0, \sin\alpha, \cos\alpha\}\,. \tag{8}$$

The initial point 0^+ is chosen to lie in the XOZ plane and the mirror normal at this point lies in the same plane, though the point 0^+ need not lie along the OZ axis. By the symmetry requirement the point 0^+ must correspond to the point 0^-, which is symmetric to 0^+ with respect to the plane YOZ. The distances from the focal points F_- and F_+ to the points 0^+ and 0^- are obviously identical. This distance is denoted by R_0. We now introduce two angles φ and ψ (see Fig. 3). The angle φ is a measure of the proximity of the foci F_+ and F_- to the XOZ plane, while ψ serves the same purpose for points 0^+ and 0^- from the plane YOZ. Specifying R_0, φ and ψ determines the positions of the foci and the initial points:

$$
\begin{aligned}
&F_+ = R_0 \sin\varphi, \quad && F_- = -R_0 \sin\varphi, && \\
&z_0^+ = R_0 \cos\varphi \cos\psi, \quad && y_0^+ = 0, \quad && x_0^+ = R_0 \cos\varphi \sin\psi, \\
&z_0^- = R_0 \cos\varphi \cos\psi, \quad && y_0^- = 0, \quad && x_0^- = -R_0 \cos\varphi \sin\psi.
\end{aligned}
\tag{9}
$$

As noted above, the normal to the surface at both 0^+ and 0^-should lie in the XOZ plane, so that the unit vector at the initial point may be characterized by a single angle. We chose this to be the angle v between the normal and the positive OX direction. Then the normal at the initial point in terms of v is

$$\vec{v}_0^+ = \{-\cos v, 0, -\sin v\}; \qquad \vec{v}_0^- = \{\cos v, 0, -\sin v\}. \tag{10}$$

As shown before, the point 0^+ corresponds to the point 1^+ on the other mirror.

The relative proximity of point 1^+ to the XOY plane is characterized by the parameter

$$\tau = z_0^+/(z_0^+ - z_1^+). \tag{11}$$

This relation determines the z-coordinate of the reference point 1^+ in terms of z_0^+ and τ:

$$z_1^+ = (1 - 1/\tau) z_0^+. \tag{12}$$

From (1), (2) and (12) one fixes the coordinates of the first reference point, the direction of the normal to the surface at the first reference point 1^+, as well as the optical path. Similarly, one determines three other points on the large mirror, 1^-, -1^+ and -1^-, together with the directions of the surface normals at these points. These are symmetry points of 1^+ with respect to the planes YOZ and XOZ. Furthermore, by repeated use of (3)–(7) and (1), (2), any number of desired reference points may be obtained. Since a real system has a finite size, the number of reference points is finite. The number of reference points will be denoted by the parameter J, which will determine the maximum number of points on each mirror.

Hence, by specifying a single dimensional parameter R_0 and six dimensionless quantities—α, φ, ψ, v, τ, J—one uniquely determines the spatial system of reference points, as well as the corresponding normal directions, on each mirror.

The system of discrete reference points thus constructed must be associated with two continuous surfaces possessing continuous first-order derivatives at each point of the given surface. This will be the case if one specifies a surface between four neighbouring reference points, passing through each, such that the normals to it equal the normals at the particular points. From considerations of symmetry the points of choice are 1^+, 1^-, -1^+ and -1^-. These points and their normals lie, pair wise, in planes which are symmetrically positioned with respect to the principal plane of symmetry. The required surface is chosen to be the surface of a paraboloid, bounded by the planes

$$
\begin{aligned}
&z = z_1, \\
&(z - z_1^\pm) N_{1\pm}^y - (y - y_1^\pm) N_{1\pm}^z = 0 \\
&(x - x_1^\pm) N_{1\pm}^z - (z - z_1^\pm) N_{1\pm}^x = 0
\end{aligned}
\tag{13}
$$

and having normals at $\pm 1^\pm$ given by

$$z = z_1 + \frac{N_x}{N_z}\left(\frac{x_1^2 - x^2}{2x_1}\right) + \frac{N_y}{N_z}\left(\frac{y_1^2 - y^2}{2y_1}\right), \tag{14}$$

where x_1, y_1, z_1 are the coordinates of 1^+, and N_x, N_y, N_z are the direction cosines of its normal.

Once the surface (14) is fixed. then by the procedure outlined by (1)–(7) one can recalculate this surface in any of the regions delimited by the four reference points, whereby the mirror surfaces thus obtained will be everywhere continuous, together with the first derivatives.

The case $J = 1$ is of particular interest: then the cycle is to be performed just once. In other words, one must design a correcting mirror to the one described in (14). The larger mirror may then be chosen to be a paraboloid of revolution, provided the following condition obtains:

$$x_1 \mid y_1 = N_x \mid N_y. \tag{15}$$

If the coordinates and the projection of the normal at the first reference point is expressed in terms of the system parameters, the condition may be restated as

$$\tau \sin \varphi = \sin \alpha[\tau + \operatorname{ctg} \psi \operatorname{tg}(2v + \psi)]. \tag{16}$$

Condition (16) establishes a relation between the system parameters and shows that the large mirror may be chosen to be a surface of revolution only if (16) has a solution.

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