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COMPUTER-SYNTHESIZED OPTICAL ELEMENTS

MODANS—OPTICAL ELEMENTS FOR ANALYSIS AND SYNTHESIS OF LASER MODE STRUCTURE

I. N. Sissakian and V. A. Soifer

Abstract—TEM mode analysers, or modans, constitute a new class of optical elements synthesized by applying computer optics technology. The early elements of this class were developed in 1982. Ways are considered to improve the technology of modan generation with an eye to expanding the functional capabilities of these elements. Analytical relationships are presented for amplitude and phase modans and supported experimentally. A more detailed consideration is given to the type of modans matched with the Laguerre–Gaussian modes and their application to optical-fibre transducers.

INTRODUCTION

Mode analysers, or modans for short, constitute a new class of optical elements formed by means of computer optics technology and used for analysis and generation of transverse laser modes. The early elements of this class were developed in 1982 for Gauss–Hermite and Laguerre–Gaussian modes [1]. They were amplitude spatial filters with constant retardation for the encoding of negative values of mode functions. These elements were successfully tested in 1983 [2]. The synthesized filters formed a standard multimode beam with a specified mode structure. Modes were separated by synthesized analysing filters for subsequent intensity measurements. Analysis was performed with the aid of spatial filters matched with corresponding mode functions.

In 1984 digital holography brought about holographic spatial filters matched with transverse modes and capable of separating modes at different angles dependent on the corresponding spatial carriers [3]. In effect, this technology evolved a new type of optical element, the amplitude modan, an analog of the amplitude diffraction grating. Unlike a diffraction grating, this element was able to separate radiation in transverse modes instead of in wavelengths.

This generation of optical elements paved the way for a fundamentally new family of spectral instruments such as analysers of transverse modes. It is worth noting that while a conventional diffraction grating is analogous to a prism which can be made by a traditional optical technology, no traditional optical element can be put in correspondence to a modan. The modan has a unique design and can be synthesized only with the aid of the technology of computer optics. Corresponding mode shaping devices were also developed.

The disadvantages of amplitude modans include a low diffraction efficiency amounting to about 0.3% in the first order of diffraction. Phase modans [4], recent newcomers to the modan family, have a diffraction efficiency about an order of magnitude higher than their amplitude counterparts.

HOLOGRAPHIC RECORDING OF TRANSVERSE MODE FUNCTIONS ON A PHYSICAL MEDIUM

Methods of digital holography proved to be suitable for the synthesis of optical elements matched with the mode beam $w(\mathbf{x})$, $x \in G$, constituted by one or more transverse modes $\psi_k \mathbf{x}$.

Amplitude modans are holograms with the transmission function

$$\Gamma_a(\mathbf{u}) = T_0 + 2\Delta T \frac{|w(\mathbf{u})|}{w_{\max}} \cos[2\pi \nu \mathbf{u} + \arg(w/\mathbf{u})], \qquad (1)$$

where

$$T_0 = A_0 + \Delta A/2, \qquad [A_0, A_0 + \Delta A] \in [0, 1],$$

$$T = \beta_a \frac{\Delta A}{4}, \quad 0 \leqslant \beta_a \leqslant 1,$$

and the amplitude transmission range

$$w_{\max} = \max w(\mathbf{u}), \quad \mathbf{u} \in D.$$

Expression (1) describes an irregular sinusoidal grating with an average period of $1/|\mathbf{v}|$.

A phase modan is obtained by a phase modulation that agrees with the complex transmission function

$$\Gamma(\mathbf{u}) = \exp\left\{ i \frac{\varphi_{\max}}{2} \beta Q\left(\frac{|w(\mathbf{u})|}{w_{\max}}\right) \cos[2\pi\mathbf{u} + \arg w(\mathbf{u})] \right\} \tag{2}$$

which corresponds to an irregular phase diffraction grating with a sinusoidal profile of lines, where $\varphi_{\max}$ is the maximum possible phase shift caused by the element (usually $\varphi_{\max} = \pi$), $\beta \in [0, 1]$ is the coefficient that determines the phase transmission range of specific element, $Q(t)$ describes a nonlinear pre-distortion chosen so as to compensate for the nonlinearity encountered in coming from amplitude to phase, and to meet the conditions $0 \leqslant Q(t) \leqslant 1$ for $0 \leqslant t \leqslant 1$, $Q(0) = 0$, and $Q(1) = 1$.

The phase modulation (2) can be achieved by both transmitting and reflecting optical elements.

A transmitting optical element is best placed in a plane $\mathbf{u} = (u, v)$ perpendicular to the optical axis. The phase relief, made of translucent material, varies in height from 0 to $\frac{\lambda}{2\Delta n} \frac{\varphi_{\max}}{\pi}$ in proportion to the phase (2), where Δn is the difference in refractive index of the material and the medium.

A reflecting optical element is placed at an angle θ to the $\mathbf{u}$ axis. The height of the reflecting phase relief varies between 0 and $\frac{\lambda}{4\cos\theta} \frac{\varphi_{\max}}{\pi}$ in proportion to the phase of $\Gamma(u_0 \cos\theta, v_0)$, where (u_0, v_0) are the Cartesian coordinates in the plane of the reflecting element. Such an element is analogous to a reflecting diffraction grating with the period $1/|\mathbf{v}| \cos\theta$ increased by the factor $1/\cos\theta$ because of the oblique incidence of the beam. The function $w(\mathbf{u})$ is assumed to vary much more slowly than the frequency.

All relations that will now be given are equally valid for both transmitting and reflecting phase types of mode optical elements described by the model (2).

Modans are to separate transverse modes by differences in angle. Holographic recording makes such a separation an easy process. Each mode function $\psi_k(\mathbf{x})$, $k = 1, 2, \ldots, L$, is recorded on the medium along with its spatial carrier frequency $\mathbf{v}_k$. One modan contains L irregular diffraction gratings. Diffraction of a mode beam at a modan causes different transverse modes to separate at different angles corresponding to the values of $\mathbf{v}_k$. The number L of mode functions that can be recorded on a holographic modan is estimated by the quantity

$$L \leqslant v^{\mu}/v^{\psi}$$

where v^{μ} is the highest spatial frequency in recording mode functions on the physical mediums, and v^{ψ} is the bandwidth of the spatial spectrum of the recorded mode functions. At present, the feasible value of L is from 20 to 30.

INTERACTION OF A MODE OPTICAL ELEMENT WITH A LIGHT BEAM

For the purpose of theoretical analysis of the mode optical element (2), we expand its transmission function by

$$\Gamma(\mathbf{u}) = \sum_{n=-\infty}^{\infty} i^n I_n\left[\frac{\varphi_{\max}}{2} \beta Q \frac{|w(\mathbf{u})|}{w_{\max}}\right] \exp\{i[n 2\pi \mathbf{v}\mathbf{u} + n \arg w(\mathbf{u})]\}, \tag{3}$$

where $I_n(\cdot)$ is Bessel's function of the nth order.

The terms of the expansion (3) correspond to different orders of diffraction at the phase diffraction grating. In this case the useful information is contained in the first order ($n = 1$):

$$\Gamma^{(1)}(\mathbf{u}) = iI_1\left[\frac{\varphi_{\max}}{2}\beta Q\left(\frac{|w(\mathbf{u})|}{w_{\max}}\right)\right]\exp[i2\pi\mathbf{v}\mathbf{u} + i\arg w(\mathbf{u})]. \tag{4}$$

The Bessel function $I_1(t)$, nonlinearly distorting w, is monotonous only on the interval $t \in [0, 1.84]$, therefore $\varphi_{\max} \leqslant 3.68$. We choose $\varphi_{\max} = \pi < 3.68$ so as to avoid using a short horizontal portion of the curve of $I_1(t)$.

To compensate for the nonlinearity of phase modulation we select the pre-distortion Q by the equation of linearization

$$I_1\frac{\varphi_{\max}}{2}\left[\beta Q\left(\frac{|w(\mathbf{u})|}{w_{\max}}\right)\right] = C|w(\mathbf{u})|, \tag{5}$$

where C is a constant.

Considering the point where $|w(\mathbf{u})|$ is a maximum, $w_{\max}$, from (5) it follows that

$$C = I_1\left(\frac{\varphi_{\max}}{2}\beta\right)\Big/ w_{\max}. \tag{6}$$

Substituting (5) into (4) yields

$$\Gamma^{(1)}(\mathbf{u}) = icw(\mathbf{u})\exp(i2\pi\mathbf{v}\mathbf{u}). \tag{7}$$

It is an easy matter to see that for the amplitude mode filter (1), $\Gamma_a^{(1)}(\mathbf{u})$ is given by (7), but with the coefficient

$$C_a = \Delta T/\omega_{\max}. \tag{8}$$

The values of C and C_a determine the proportion of light fields passing through the optical elements.

ESTIMATION OF ENERGY EFFICIENCY OF MODE OPTICAL ELEMENTS

Because mode optical elements are analogous to diffraction gratings, only a fraction ε_1 of the light incident on the element, ε_{in}, is directed in the first diffraction order. To estimate ε_1 we assume that the optical mode element is illuminated by a plane normally incident wave and recorded by a photodetector with large aperture D_1 in the image plane. For a plane illuminating wave we have

$$\frac{\varepsilon_1}{\varepsilon_{\text{in}}} = \frac{1}{|D|}\int_D |\Gamma^{(1)}(\mathbf{u})|^2\,\mathrm{d}^2\mathbf{u}, \tag{9}$$

$$\frac{\varepsilon}{\varepsilon_{\text{in}}} = \frac{1}{|D|}\int_D |\Gamma(\mathbf{u})|^2\,\mathrm{d}^2\mathbf{u},$$

where

$$|D| = \int_D \mathrm{d}^2\mathbf{u},$$

ε is the light transmitted by the optical element. For the phase optical element, from (2) and (7) we obtain

$$\varepsilon/\varepsilon_{\text{in}} = 1, \tag{10}$$

$$\varepsilon/\varepsilon_{\text{in}} = C^2\bar{\omega}^2, \tag{11}$$

where

$$\bar{\omega}^2 = \frac{1}{|D|}\int_D |w(\mathbf{u})|^2\,\mathrm{d}^2\mathbf{u}. \tag{12}$$

For amplitude spatial filters, from (1) and (7) it follows in a similar way

$$\varepsilon_a/\varepsilon_{in} = T_a^2 + 2C_a^2\bar{w}^2, \quad (13)$$

$$\varepsilon_{a_1}/\varepsilon_{in} = C_a^2\bar{w}^2. \quad (14)$$

Comparing (11) and (14) yields a relation for energy efficiencies of the phase and amplitude filters

$$\frac{\varepsilon_1}{\varepsilon_{a_1}} = \frac{C^2}{C_a^2} = \left[\frac{I_1\left(\frac{\varphi_{max}}{2}\beta\right)}{\frac{\Delta A}{4}\beta_a}\right]^2. \quad (15)$$

Note that at $w(\mathbf{u}) = \text{const}$ the formulae (10)–(15) become the familiar relations [5] for diffraction efficiency of sine amplitude and phase gratings

$$\frac{\varepsilon_{a_1,gr}}{\varepsilon_{in}} = \left(\frac{\Delta A}{4}\beta_a\right)^2; \qquad \frac{\varepsilon_{gr}}{\varepsilon_{in}} = I_1^2\left(\frac{\varphi_{max}}{2}\beta\right), \quad (16)$$

where modulation is taken the same as for the mode filters.

Analysis of (15) and (16) lead us to draw an important conclusion that the phase optical elements ensure the same energy gain over the amplitude elements as a phase diffraction grating does over its amplitude counterpart.

OPTICAL ELEMENTS MATCHED WITH LAGUERRE–GAUSSIAN MODES

For Laguerre–Gaussian modes [6] typical of gradient lightguides with a square-law refractive index, and spherical mirror resonators, simple formulae can be obtained for all the above quantities.

A rapidly decreasing Gaussian factor enables the functions $\psi_{pl}(\mathbf{x})$ to be considered in a limited domain D of the size of several beam waist radii σ. Let D be a circle of diameter d.

Using the formulae for the Laguerre–Gaussian complex amplitude [6], asymptotic approximations of Laguerre polynomials, and numerical evaluations we obtain the following relations:

$$\frac{\varepsilon_1}{\varepsilon_{in}} = I_1^2\left(\frac{\varphi_{max}}{2}\right)\frac{1}{2}\left(\frac{\sigma}{d/2}\right)^2, \qquad \frac{\varepsilon}{\varepsilon_{in}} = 1, \quad (17)$$

$$\frac{\varepsilon_{a_1}}{\varepsilon_{in}} = \left(\frac{\Delta A}{4}\right)^2\frac{1}{2}\left(\frac{\sigma}{d/2}\right)^2, \qquad \frac{\varepsilon}{\varepsilon_{in}} = T_0^2 + \left(\frac{\varepsilon_{a_1}}{\varepsilon_{in}}\right)^2, \quad (18)$$

$$\frac{\varepsilon_{1,gr}}{\varepsilon_{in}} = I_1^2\left(\frac{\varphi_{max}}{2}\right), \qquad \frac{\varepsilon_{a_1gr}}{\varepsilon_{inc}} = \left(\frac{\Delta A}{4}\right)^2, \quad (19)$$

$$\frac{C^2}{C_a^2} = \left[\frac{4I_1(\varphi_{max}/2)}{\Delta A}\right]^2. \quad (20)$$

Here, the quantity C^2/C_a^2 (20) is the energy gain of the phase optical elements over the amplitude elements achieved in energy efficiency $\varepsilon_1/\varepsilon_{a_1}$, in the light passing through the shaping element, and in the scale of measured mode powers in the mode analyser.

By way of illustration, for $\sigma = 0.65$ mm, $d = 5$ mm, $\varphi_{max} = \pi$, $A_0 = 0.1$, $\Delta A = 0.7$, $l = 0$, we obtain $C^2/C_a^2 = 10.66$.

Because the Laguerre–Gaussian mode functions decrease rapidly, the useful light field in the first diffraction order is formed only by the central portion of mode optical elements which is characterized by the deepest line modulation of a sinusoidal diffraction grating. Therefore, mode optical elements are sensitive to such factors as nonuniformity of the illuminating beam $E/\mathbf{u}$ and nonlinear response in sinusoidal amplitude and phase transmission.

EXPERIMENTAL RESULTS

Amplitude and phase holographic modans were implemented as transmitting and reflecting optical elements. Experimental testing demonstrated their serviceability and the matching between the performance and specified characteristics [4]. These modans permitted various experiments with graded-index fibre lightguides. Specifically, a light fibre with refractive index law close to a parabolic profile and supporting Laguerre–Gaussian modes was investigated [7]. The powers of four modes were measured as functions of the radius of the axial excitation beam. The results demonstrated a sufficiently close agreement with the theoretical findings [8].

The modans were also used to separate the modes at the output of a graded-index optical fibre subjected to periodic microbends [9]. Mode coupling as a function of bend amplitude was investigated. The results proved useful in the synthesis of sensitive acousto-optical transducers and devices based on multimode graded-index fibres.

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