Computer Optics Vol. 2, No. 2, pp. 167–171, 1990
Printed in Great Britain

0955–355X/90 $10.00 + 0.00

MODELLING THE INTERFERENCE OF SURFACE AND SPATIAL WAVES

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Abstract—The problem of interference of a metallic surface EM wave (SEMW) and the spatial radiation launched in SEMW excitation is discussed within the impedance approximation.The solution of the parabolic equation for the field above the metal is obtained in terms of special functions. The wave emanating from the metal edge is computed by the Kirchhoff–Huygens diffraction integral. The interference pattern is observed at some distance from the metal specimen in a plane perpendicular to the metal surface. The distances between the extrema of this pattern agree with values obtained in the geometrical optics approximation for two interfering rays, the ray emanating from the metal edge and the spatial ray formed at the slit at which the surface wave is launched. The phase difference between the two rays is controlled by the aperture launching the surface wave, the parameters of the metal specimen and the length of the metal surface. The interference parameters yield the real part of the metal permittivity. This model was used to explain the experimental data for a film of gold deposited on a glass plate in vacuum. Combining the real part of the dielectric constant and the imaginary part determined by measuring the surface wave propagation path gives information on the surface of the metal specimen.

The surface electromagnetic wave (SEMW) is a sensitive tool in sounding out the properties of the subsurface layers of metals. The amplitude of the sounding wave is largest at the surface and by measuring its attenuation one may obtain the absorption coefficient in the metallic specimen [1] and deduce the imaginary part of the complex effective index of refraction. The real part of the effective index of refraction can be determined by phase spectroscopy of surface waves [2], specifically, from interference measurements of the phase shift.

The idea of the experiment is illustrated in Fig. 1. The exciting radiation is incident on the aperture formed between the metal and a screen, is diffracted at this slit, and is partially converted into a surface wave. This SEMW propagates for a distance a along the surface of the metal to its edge where it becomes a spatial wave that interferes with the wave diffracted at the slit. The interference pattern is recorded in a plane perpendicular to the metal surface and at a distance b from the edge of the metallic specimen.

We analyse the intensities in the interference pattern in two ways. First there is the problem of propagation of the surface wave and the diffracted radiation over the metal, using the impedance approximation. The surface wave propagates along the surface $z = 0$, characterized by the impedance

$$Z = R - iX = \varepsilon^{-1/2}, \quad (R, X > 0, \tag{1}$$

where ε is the permittivity of the medium.

We consider TM polarized monochromatic radiation at frequency ν. The non-zero components are H_y, E_x and E_z. Evaluation of the field reduces to the determination of H_y, while E_x and E_z are calculated by Maxwell's equations.

The surface wave is excited at a slit of height d in the plane $x = 0$. The problem is invariant under translations along the y-axis. In the domain of SEMW propagation we are to solve the wave equation

$$H(x, z, t) = 0 \tag{2}$$

subject to the boundary condition of Leontovich [3] for media of large ε, *viz.*

$$\frac{\partial H}{\partial z} + ikZH = 0 \qquad \text{for} \quad z = 0,\ k = 2\pi\nu. \tag{3}$$

The initial condition is given by the exciting field at the slit

$$V_0(z) = A\,e^{-ik\alpha z} + B\,e^{ik\alpha z} \qquad \text{for} \quad 0 < z < d. \tag{4}$$

We seek the solution in the form

$$H(x, z, t) = V(x, z)\,e^{ikx - i\omega t}. \tag{5}$$

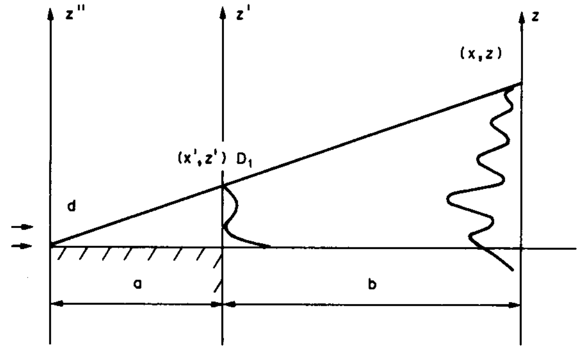

Fig. 1. Generation of interference pattern under aperture excitation of surface waves.

Then Eq. (2) reduces to a diffusion equation with the coefficient $D = i/2k$

$$\frac{\partial V}{\partial x} = D \frac{\partial^2 V}{\partial z^2}, \tag{6}$$

and the boundary condition becomes

$$\frac{\partial V}{\partial z} + ikZV(x, z) = 0 \qquad \text{at} \quad z = 0. \tag{7}$$

The formalism of solution of the parabolic equation (6) is described in detail elsewhere [3]. The solution may be written

$$V(x, z) = \int_{-\infty}^{\infty} dz' G(x, z - z') V_0(z'), \tag{8}$$

where

$$G(x, z - z') = \left(\frac{k}{2\pi x}\right)^{1/2} \exp \frac{ik(z - z')^2}{2x} - \frac{i\pi}{4}$$

is the Green function, and $V_0(z')$ is the initial condition that is continued into the region $z < 0$ in an appropriate manner taking into account the boundary condition.

For the initial condition (4) the continued initial condition has the form

$$V_0(z) = \begin{cases} 0, & z > d \\ A\,e^{-ik\alpha z} + B\,e^{ik\alpha z}, & 0 < z < d \\ A\dfrac{\alpha - Z}{\alpha + Z}\,e^{ik\alpha z} - B\dfrac{\alpha + Z}{\alpha - Z}\,e^{-ik\alpha z} & \\ \quad + 2Z\left(\dfrac{A}{\alpha + Z} - \dfrac{B}{\alpha - Z}\right)e^{-ikZz}, & -d < z < 0 \\ 2Z\left(A\dfrac{1 - e^{-ik(\alpha + Z)d}}{\alpha + Z} - B\dfrac{1 - e^{ik(\alpha - Z)d}}{\alpha - Z}\right)e^{-ikZz}, & z < -d. \end{cases} \tag{9}$$

Substituting (9) into (8) we get the solution of the parabolic equation in the form

$$V(x, z) = \frac{1}{2}e^{(\tau - \zeta)^2} A\,e^{-2\rho\zeta} W(\rho + \tau - \zeta) + B\,e^{2\rho\zeta} W(-\rho + \tau - \zeta)$$

$$+ \frac{1}{2}e^{(\tau + \zeta)^2}\left(\frac{B\,e^{2\rho\zeta}}{\rho - \sigma} - \frac{A\,e^{-2\rho\zeta}}{\rho + \sigma}\right) 2\sigma W(\sigma + \tau + \zeta)$$

$$- A\,e^{-2\rho\zeta}\frac{\rho - \sigma}{\rho + \sigma} W(-\rho + \tau + \zeta) - B\,e^{2\rho\zeta}\frac{\rho + \sigma}{\rho - \sigma} W(\rho + \tau + \zeta)$$

$$+\frac{1}{2}e^{\tau^2}\left(\frac{A}{\rho+\sigma}-\frac{B}{\rho-\sigma}\right)2\sigma W(\sigma+\tau)$$

$$+\left(B\frac{\rho+\sigma}{\rho-\sigma}-A\right)W(\rho+\tau)+\left(A\frac{\rho-\sigma}{\rho+\sigma}-B\right)W(-\rho+\tau), \tag{10}$$

where

$$\tau=\exp(i\pi/4)\frac{z}{x}\sqrt{\frac{kx}{2}},$$
$$\sigma=\exp(i\pi/4)Z\sqrt{kx/2},$$
$$\rho=\exp(i\pi/4)\alpha\sqrt{kx/2},$$
$$\zeta=\exp(i\pi/4)\frac{d}{x}\sqrt{\frac{kx}{s}},$$

and

$W(\xi)=\exp(\xi^2)\frac{2i}{\sqrt{\pi}}\int_{i\infty}^{\xi}\exp(\eta^2)\,d\eta$ is the integral function of the complex argument.

Suppose that the slit is irradiated by a plane wave, then $A=1$, $B=0$, and $\alpha=0$ in Eq. (4), so that the solution may be written as

$$V(x,z)=\frac{1}{2}e^{(\tau-\zeta)^2}W(\tau-\zeta)+e^{(\tau+\zeta)^2}-W(\sigma+\tau+\zeta)+\frac{1}{2}W(\tau+\zeta)+e^{\tau^2}W(\sigma+\tau)-W(\tau). \tag{11}$$

This solution (obtained under the impedance approximation) is valid for x obeying the inequality

$$|kx/\varepsilon^2|\ll\pi. \tag{12}$$

It is valid in experiments with metal specimens of $\varepsilon\sim10^3$ and $x\sim10$ cm in the range of $\nu\sim1000\ \mathrm{cm}^{-1}$.

A calculation of the field near the metal surface was carried out for a metal specimen with $\nu_p=80{,}000\ \mathrm{cm}^{-1}$ and $\nu_\tau=800\ \mathrm{cm}^{-1}$, aslit of height $d=60\ \mu m$ irradiated by a frequency of $\nu=1000\ \mathrm{cm}^{-1}$.

The impedance of the metal specimen is calculated as

$$X=\frac{\nu}{\sqrt{2}\,\nu_p}\left[\left(1+\frac{\nu_\tau^2}{\nu^2}\right)^{1/2}+1\right]^{1/2}$$
$$R=\frac{\nu}{\sqrt{2}\,\nu_p}\left[\left(1+\frac{\nu_\tau^2}{\nu^2}\right)^{1/2}-1\right]^{1/2}. \tag{13}$$

The distribution of the field derived by Eq. (11) is similar to that describing the Fraunhofer diffraction at a $2d$ slit, so that the directions to the minima θ_l obey the condition

$$\theta_l=\lambda l/2d,$$

where $l=1,2$, etc. and λ is the wavelength.

The presence of the impedance plane, however, causes the zero diffraction maximum to be divided into a surface wave and an edge spatial wave. Figure 2 shows distributions of the field near the surface recorded for different distances $x=a$ of the exciting aperture. The surface wave is localized at the metal surface and dies out in its propagation along the surface. The maximum of the spatial radiation finds itself elevated from the surface with the distance a, and the minimum separating this radiation from the surface wave is near the surface. If we follow the phase of SEMW we note that if differs from the phase of the surface wave that propagates along the surface without being affected by the spatial diffracted radiation. The additional phase shift is controlled by the size of the exciting aperture d, the parameters of the metal and the distance a from the aperture. However, it may be deemed constant in a certain interval of distance a.

If we calculate the field at the edge of the specimen, we can obtain the distribution of intensity along the z-axis of the interference pattern by making use of the Huygens principle. We seek the

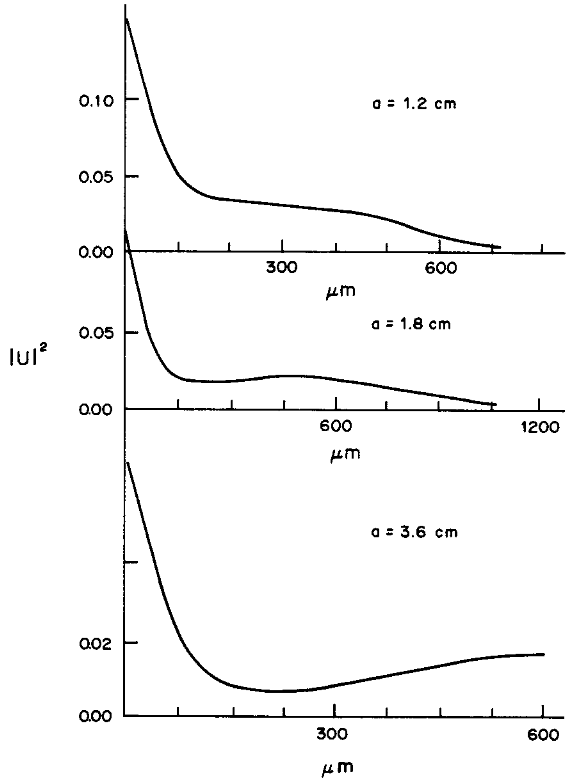

Fig. 2. Distribution of intensity near the surface in the vertical coordinate for three distances $x = a$ from the exciting slit.

field as the Kirchhoff integral [4]

$$H_y(x, z) = \frac{1}{\sqrt{8\pi k}} \exp\left(\frac{i\pi}{4}\right) \int_C \mathrm{d}c \left[\frac{H_y}{n} \frac{\mathrm{e}^{-ikr}}{\sqrt{r}} - H_y \frac{\partial}{\partial n} \frac{\mathrm{e}^{-ik}}{r}\right], \tag{14}$$

where C is any contour embracing the point (x, z), and r is the distance from this point to the contour. Let us choose the contour passing along the z'-axis and closing at infinity. Assuming that $kr \gg 1$, the integral reduces to the form

$$H_y(x, z) = \sqrt{\frac{k}{8\pi}} \exp\left(i\frac{\pi}{4} + ika\right) \int_0^{D_m} \mathrm{d}z' \frac{\mathrm{e}^{-ikr}}{r} V(x, z')\left(\frac{a}{\sqrt{a^2 + z'^2}} + \frac{b}{r}\right), \tag{15}$$

where $V(x, z')$ is the field at a distance a from the slit launching the surface wave calculated by Eq. (11), and D_m the coordinate of the diffraction minimum. For $z' > D_m$, the field is assumed to be zero.

Figure 3 presents interferograms calculated by Eq. (15) for three different values of a and with the parameters v_p and v_τ as indicated above.

The analysis suggests that the positions of extrema of interferograms may be determined by the geometrical optics approximation, that is, the equation

$$n'_{\mathrm{eff}} a + \sqrt{b^2 + z_m^2} - \sqrt{(b + a)^2 + z_m^2} = (m + \Delta m)/2v \tag{16}$$

holds true, where n'_{eff} is the real part of the effective refractive index for SEMW, m is the number of an extremum—even for maxima and odd for minima, and Δm is the additional phase of SEMW.

The theoretical interferograms were compared with the experimental interferograms obtained for a film of gold, about 2000 Å thick, that had been thermally deposited onto a polished glass plate. We used the experimental distributions of intensity in the interference patterns recorded for different a at the same frequency $v = 984.4\ \mathrm{cm}^{-1}$ to determine the real part of the effective refractive index for SEMW, and by measuring the attenuation of SEMW we determined the imaginary part

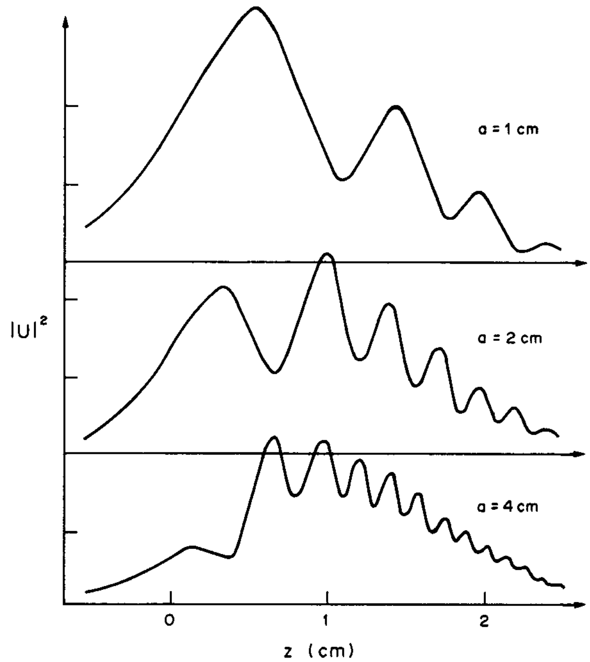

Fig. 3. Interferograms computed by Eq. (16) for different a.

Table 1.

	No. of extremum 2	3	4	5	6	7	8	9	10
$a = 1.4$ cm and $b = 43.2$ cm									
Theor.	1.61	1.97	2.32	2.57	2.88	3.11	3.32	3.51	3.72
Exp.	1.61	1.97	2.32	2.61	2.85	3.10	3.51	3.53	3.71
$a = 2.5$ cm and $b = 43.2$ cm									
Theor.	1.08	1.38	1.60	1.85	2.06	2.25	2.41	2.58	2.76
Exp.	1.08	1.37	1.68	1.87	2.07	2.27	2.43	2.59	2.75

of the index [1]. The following parameters of electrons were obtained: $\nu_p = 51{,}000\ \text{cm}^{-1}$ and $\nu_\tau = 460\ \text{cm}^{-1}$. Calculations of interference patterns for these values were carried out by Eq. (16). The values of extrema of both experimental and theoretical interferograms are summarized in Table 1 for two sets of values of a and b (see Fig. 1).

The calculated values are seen to coincide well with the experimental interferograms. This proves that the treatment of interferograms in the geometrical optics approximation was correct. However, the calculation by the impedance approximation can supply information on both the real and on the imaginary part of the effective index of refraction of SEMW.

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