[20] Algorithms of linear spectral mixture analysis for hyperspectral images using Base Map

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Abstract

The authors propose two algorithms of linear spectral mixture analysis for hyperspectral images using base map. Base map data are used to refine coefficients of spectral mixture on the edges of map objects (in the first algorithm) or acquire spectral signatures of small objects (in the second algorithm), that does not occupy any pixel on input image entirely. The set of mixed signatures may be already known with undefined coefficients or unknown with extraction on one of the stages of the algorithm.

Keywords: hyperspectral images, spectral unmixing, hyperspectral analysis, subpixel selection, least squares method, base map.

Citation: Denisova A.Yu. Algorithms of linear spectral mixture analysis for hyperspectral images using Base Map / Denisova A.Yu., Myasnikov V.V. // Computer Optics. – 2014. – Vol. 38(2). – P. 297-303.

Introduction

Geographic information systems are now widely used to maintain current base map data at any resolution using various data sources. As a result, databases of geographic information systems contain large amounts of graphic and semantic information, which can be used as data for many applications, including the data analysis of the Earth remote sensing (ERS). This paper offers several ways to use base map data contained in geographic information systems to perform the problems of hyperspectral images analysis.

The linear spectral unmixing problem is one of the key issues in terms of analysis of hyperspectral ERS data. It is assumed that each pixel of the image is described by a linear model for the spectral mixture of some spectral signatures [1-3], and the linear spectral unmixing problem is to find coefficients of this linear combination.

The set of methods of Linear Spectral Analysis (LSMA) was developed and systematized in papers written by Professor C.I. Chang, who had devoted his three monographs [1-3] to the issues of hyperspectral image processing. A slightly different classification is presented in paper [4]. Following the classification proposed in [1-3], the methods can be divided, according to the usage degree for a priori information, into *Supervised* LSMA (*SLSMA*), when the list of unmixing signatures is known, and *Unsupervised* LSMA (*ULSMA*), when no priori information about the list of signatures is available. The ULSMA specific feature is the valuation strategy for the set of signatures composing the image in some optimal manner followed by application of one of the SLSMA algorithms. The SLSMA algorithms are a combination of data modifying methods to account for the diverse priori information about interdependencies of channels and signatures, and linear spectral separation methods for mixtures. The latter ones are divided, according to the available constraints, into the unmixing coefficients and are based on appropriate optimization methods.

In this paper we consider the same problems, but they are added in wording with additional information available on the base map in geographic information systems. Using the base map containing the information about the spatial arrangement of real objects (including small ones), some additional constraints may be imposed on linear spectral unmixing problem, and significant improvements may be obtained as solutions both on the edges of map objects and when small objects available (which do not contain any pixel on input image entirely). The proposed approach allows to implement the so-called subpixel selection method when the derivable spectral coefficients, or the spectral signatures, correspond to the areas the physical dimension of which is less than the linear resolution of hyperspectral imaging.

The work has been organized as follows. The first section presents the spectral unmixing problem of linear signature mixture and provides the information about the constraints.

The following sections contain new results: the second section describes the linear spectral unmixing algorithm that enables make the spectral separation on the edges of map areas using the base map; the third section presents the subpixel spectral selection algorithm that enables obtain spectral signatures of small map objects the dimensions of which may be smaller than image readings. The experimental results of the spectral unmixing algorithm are given in the fourth section, whereas the subpixel spectral selection algorithm – in the fifth section.

Conclusions, acknowledgments and references are given at the end of the paper.

1. Spectral unmixing problems in linear mixture of spectral signatures

To describe pixel $\bar{v}$ of the image we use a linear model of spectral mixture [1-3, 4], when the source pixel is represented as a linear combination of several spectral signatures $M=\left(\bar{m}_1,\ldots,\bar{m}_p\right)$

$$\bar{v}=M\bar{\alpha}+\bar{n} \tag{1}$$

where $\bar{n}$ is an error of the model and measurements $\bar{\alpha}^T=\left(\alpha_1,\ldots,\alpha_p\right)^T$ are coefficients which satisfy one or both constraints:

1) normalization

$$\sum_{j=1}^{p}\alpha_j=1 \tag{2}$$

2) non-negativity

$$\alpha_j\geq 0, 0\leq j\leq p \tag{3}$$

The coefficients are searched by minimizing the value of the root-mean-square deviation of the linear mixture of spectral signatures from the true value of hyperspectral pixel:

$$\varepsilon^2=\left(\bar{v}-M\bar{\alpha}\right)^T\left(\bar{v}-M\bar{\alpha}\right)\to\min_{\bar{\alpha}} \tag{4}$$

Sometimes, to account additional factors the problem is set in the following way:

$$\varepsilon^2=\left(\bar{v}-M\bar{\alpha}\right)^T A\left(\bar{v}-M\bar{\alpha}\right)\to\min_{\bar{\alpha}} \tag{5}$$

where A is a weighting matrix that takes into account the errors in each channel and their interconnection. Without loss of generality, we shall assume that A is an identity matrix; in all other cases the problem (5), by means of linear transformations of the matrix of spectral signatures and pixel-vectors, may be reduced to the problem (4). The methods and algorithms for solving the problem (4) are presented both in the given reference papers [1-4] and in specialized studies [5-8].

2. Spectral unmixing algorithm using the base map

The priori information which is taken into consideration in the spectral unmixing problem based on the base map data is assumed to be as follows:

1) a list of the known spectral signatures *LS,* dimension *NS* $\left\{\bar{s}_i\right\}_{i=0}^{NS-1}$ (not necessarily completed),

2) a list of the known types of areas/map objects in a digital vector map *LR,* dimension *NR*,

3) a compliance matrix of δ signatures and map areas with a dimension of *NS*×*NR,* respectively. The value of each matrix element (i_{LS},i_{LR}) is defined as follows:

- $\delta\left(i_{LS},i_{LR}\right)=2$ if the signature i_{LS} may be present in the spectral mixture for the area i_{LR};
- $\delta\left(i_{LS},i_{LR}\right)=-2$, if the signature i_{LS} is not present in the spectral mixture for the area i_{LR};
- $\delta(i_{LS},i_{LR})\in(0,1)$ if the signature i_{LS} is used with a proper coefficient.

The set of these data may be stored as a database and used for a variety of applications.

The inputs for the specific problem of spectral unmixing are:

1) georeferenced hyperspectral image $\bar{v}\left(n1,n2\right)$ with a resolution *R,* dimension *N1*×*N2*;

2) the “masks” of areas / objects, each of which corresponds to the index of *LR*. The “masks” of areas can be either vector– or raster-typed obtained by means of GIS. In this latter case, the mask resolution should be several times higher than *R* (resolution of the input image). The areas may not intercross, and the dimension of each area should be greater than the amount of image reading;

3) the ExtendSpectrum parameter. If its value is the “Truth”, the list of signatures in solving the problem may be supplemented. The supplemented list will be called *LSE*, the dimension list *NSE* will be *NSE*≥*NS*. If the parameter value is the “False”, then the lists *LSE* and *LS* will coincide;

4) the optional parameter EPS intended to stop the supplement procedure for the signature list.

It is recognized that the geo-referencing for the input image has been accurately performed. If the data about spectral signatures in the image are completely missing, then, to initialize the signatures list it is proposed to use any of the known Lookup Methods of “pure” pixels, e.g. the algorithm N-FINDR [9].

The outputs of the proposed algorithm are:

1) the supplemented list of signatures *LSE*;

2) *NSE* is a channel image, dimension *N1*×*N2*, containing, in each reading, the reference coefficients of the corresponding (by its position) hyperspectral reading

of the source image in the form of the spectral signatures mixture *LSE*. Each channel of output image $\lambda_i(n1,n2)$ corresponds to the set of coefficients for the spectrum signature with a number of i from the list of *LSE*;

3) the remaining hyperspectral images $x(n1,n2)$ of the referenced hyperspectral reading of the source image with the linear mixture of spectral signatures.

A step-by-step description of the proposed spectral unmixing algorithm using the base map data is given below.

1. Among the set **V** of the readings of the complete image, the readings fully laid within the ''mask'' areas (not on the edges) are selected. Let us indicate these varieties of readings as $\left\{\mathbf{V}_j\right\}_{j=0}^{NR-1}$.

2. For each reading of the set $\mathbf{V}_j$ the problem of linear spectral unmixing (4) is solved under the constraints (2) and (3). As a result, the proportion of each individual spectral component is determined from the variety *LS*, for which $\delta(i_{LR},i_{LS})$ allows for the presence (if the share is fixed, it is also preliminary fixed in the corresponding system, and not being the solution to this problem). Solving this problem results to the values $\lambda_i(n1,n2)$ for the respective indices i from *LS* and the readings from $\mathbf{V}_j$.

3. There is formed the set $\mathbf{X}=\bigcup_{j=0}^{NR-1}\mathbf{V}_j$ of readings which contains hyperspectral remains obtained in step 2 of the spectral unmixing.

$$x(n1,n2)\equiv\overline{v}(n1,n2)-\sum_{i=0}^{NS-1}\lambda_i(n1,n2)\overline{s}_i$$

4. If ExtendSpectrum = the ''truth'', steps 4.1 through 4.3 are to be executed. Otherwise, there will be a transition to step 5.

4.1. For the set **X** of hyperspectral remains a search procedure of "pure" pixels with the EPS parameter is performed. If the original list of signatures was not complete, the set of remains will contain linear combinations of missing signatures. Then, the "pure" hyperspectral remains will indicate the spectral signatures from the set **V** to be included into the list of *LSE*.

4.2. For the set $\left\{\mathbf{V}_j\right\}_{j=0}^{NR-1}$ the application (4) is to be solved under the constraints (2)-(3) to supplement the list of signatures. Let us denote the resulting spectral coefficients as $\lambda_i(n1,n2)$, where i is the index from the list of signatures.

4.3. For the readings from $\left\{\mathbf{V}_j\right\}_{j=0}^{NR-1}$ the values of hyperspectral remains will be recalculated according to the supplemented list of signatures:

$$x(n1,n2)\equiv\overline{v}(n1,n2)-\sum_{i=0}^{NSE-1}\lambda_i(n1,n2)\overline{s}_i$$

5. For each 'area-signature' pair there is determined a distribution law for the spectral coefficients $\left\{\lambda_i(n1,n2)\right\}_{(n1,n2)\in V_j}$.

Let us denote the corresponding distribution (probability density) laws:

$\left\{p_{ij}(\lambda)\right\}_{i=\overline{0,NSE-1},\ j=\overline{0,NR-1}}$.

In the case of the normal distribution it is sufficient to determine the mathematical expectation and variance ratios.

6. There is formed the set of pixels $\mathbf{V}^*=\mathbf{V}\setminus\bigcup_{j=0}^{NR-1}\mathbf{V}_j$ reaching the edges of map areas/objects.

7. For each pixel $(n1, n1)$ from the list $\mathbf{V}^*$ the area proportions occupied by a particular area is to be calculated. Let us denote the areas:

$\left\{S_j(n1,n2)\right\}_{j=0}^{NR-1}$.

Obviously, the constraint $\sum_{j=0}^{NR-1}S_j(n1,n2)=1$ should be fulfilled.

8. For each reading $(n1, n1)$ from the list $\mathbf{V}^*$ (located on the edge) the following values are determined with the signature $\overline{v}(n1,n2)$:

$$\lambda_i(n1,n2)=\sum_{j=0}^{NR-1}S_j(n1,n2)\lambda_{ij}(n1,n2)$$

as solutions for the following application task:

$$\begin{cases}\alpha\left(\overline{v}(n1,n2)-\sum_{i=0}^{NSE-1}\sum_{j=0}^{NR-1}\lambda_{ij}(n1,n2)S_j(n1,n2)\overline{s}_i\right)^2-\\ -(1-\alpha)\sum_{i=0}^{NSE-1}\sum_{j=0}^{NR-1}\ln p_{ij}\left(\lambda_{ij}(n1,n2)\right)\to\min\limits_{\left\{\lambda_{ij}(n1,n2)\right\}_{i=\overline{0,NSE-1},\ j=\overline{0,NR-1}}}\\ \lambda_{ij}(n1,n2)\geq 0,\qquad i=\overline{0,\ldots NSE-1},\ j=\overline{0,\ldots NR-1};\\ \sum_{i=0}^{NSE-1}\lambda_{ij}(n1,n2)=1,\quad j=0..NR-1.\end{cases}$$

Here $\alpha\in[0,1]$ is a parameter which characterizes the relative value of each of the summands in the criterion objective function. For the case when probability distributions

$\left\{p_{ij}(\lambda)\right\}_{i=\overline{0,NSE-1},\ j=\overline{0,NR-1}}$

are normal, the above criterion will be as follows:

$$\begin{cases} \alpha\left(\bar{v}\left(n1,n2\right)-\sum_{i=0}^{NSE-1}\sum_{j=0}^{NR-1}\lambda_{ij}\left(n1,n2\right)S_j\left(n1,n2\right)\bar{s}_i\right)^2+ \\ +\left(1-\alpha\right)\sum_{i=0}^{NSE-1}\sum_{j=0}^{NR-1}\frac{\left(\lambda_{ij}\left(n1,n2\right)-m_{ij}\right)^2}{\sigma_{ij}^2}\to\min_{\left\{\lambda_{ij}\left(n1,n2\right)\right\}_{i=\overline{0,NSE-1},\ j=\overline{0,NR-1}}} \\ \lambda_{ij}\left(n1,n2\right)\geq 0, \qquad i=\overline{0,\ldots NSE-1},\ j=\overline{0,\ldots NR-1}; \\ \sum_{i=0}^{NSE-1}\lambda_{ij}\left(n1,n2\right)=1, \quad j=\overline{0,\ldots NR-1}. \end{cases}$$

9. For each reading located on the edge the hyperspectral values of remains will be recalculated as follows:

$$x\left(n1,n2\right)\equiv\bar{v}\left(n1,n2\right)-\sum_{i=0}^{NSE-1}\lambda_i\left(n1,n2\right)\bar{s}_i.$$

Note. Any extract methods for spectral pure elements can be used as the list supplement procedure in step 4.1, e.g. the algorithm N-FINDR [9]. The value and the meaning of parameter EPS are determined by a particular algorithm resulting to the stop of the list supplement procedure. When using the algorithm N-FINDR, the value EPS characterizes a threshold value which finally determines the number of selected signatures which serve as the so-called "pure" pixels. The connection of the threshold value EPS with the number of selected signatures is related by limiting the amount of the eigen values of the correlation matrix of image channels (using the Karhunen-Loeve expansion).

3. Spectral selection algorithm using the base map

Requirements to the *priori information* necessary for spectral selection of the signature of small objects coincide with the above mentioned requirements valid for the spectral unmixing algorithm.

The inputs for the particular problem to the signature spectral selection of small objects are as follows:

1-4) input data which coincide with the respective positions of the input data valid for the spectral unmixing algorithm using the base map outlined in the previous section;

5) the index t for the small map area/object (this area does not contain any pixel on input image entirely) for which it is required to determine the spectral signature.

The outputs of the algorithm are:

1) the supplemented list of signatures *LSE*;

2) the spectral signature $\bar{s}$ for the area *t*.

A step-by-step *description of the spectral selection algorithm* of the signature of the small area/object using the base map is given below.

1-7. The relevant steps of the spectral unmixing algorithm are implemented using the base map as described in the previous section, for those areas which have at least one whole reading in their composition .

8. For the specified index t of the "small" area there is formed a subset $\mathbf{V}^{**}$ of the set $\mathbf{V}^{*}$ of readings (a definition of this set is given in the previous algorithm), which ''contain'' this area.

9. The target spectral signature $\bar{s}$ of the field t is defined as the solution to the following optimization problem:

$$\begin{cases} \alpha\sum_{(n1,n2)\in V^{**}}\left(\begin{array}{l}\bar{v}\left(n1,n2\right)-S_t\left(n1,n2\right)\bar{s}- \\ -\sum_{j=0,j\neq t}^{NR-1}\sum_{i=0}^{NSE-1}\lambda_{ij}\left(n1,n2\right)S_j\left(n1,n2\right)\bar{s}_i\end{array}\right)^2- \\ -\left(1-\alpha\right)\sum_{j=0,j\neq t}^{NR-1}\sum_{i=0}^{NSE-1}\ln p_{ij}\left(\lambda_{ij}\left(n1,n2\right)\right)\to\min_{\bar{s},\left\{\lambda_{ij}\left(n1,n2\right)\right\}_{i=\overline{0,NSE-1},\ j=\overline{0,NR-1}}} \\ \lambda_{ij}\left(n1,n2\right)\geq 0, \qquad i=\overline{0,\ldots NSE-1},\ j=\overline{0,\ldots NR-1}; \\ \sum_{i=0}^{NSE-1}\lambda_{ij}\left(n1,n2\right)\leq 1, \quad j=\overline{0,\ldots NR-1}. \end{cases}$$

Here $\alpha\in[0,1]$ is a parameter which characterizes the relative value of each of the summands in the criterion objective function. If $\alpha\to 0$, when spectral coefficients are determined from the condition $\lambda_{ij}=\lambda_{ij}^*\equiv\arg\max_{\lambda} p_{ij}\left(\lambda\right)$, the explicit formula for the derived target area spectral signature will be as follows:

$$\bar{s}=\frac{\sum_{(n1,n2)\in V^{**}}S_t\left(n1,n2\right)\left(\bar{v}\left(n1,n2\right)-\sum_{j=0,j\neq t}^{NR-1}S_j\left(n1,n2\right)\sum_{i=0}^{NSE-1}\lambda_{ij}^*\bar{s}_i\right)}{\sum_{(n1,n2)\in V^{**}}\left(S_t\left(n1,n2\right)\right)^2}$$

For a normal distribution model the value

$$\lambda_{ij}^*\equiv\arg\max_{\lambda}\ p_{ij}\left(\lambda\right)$$

is determined as the average unmixing coefficient of the *j*-typed area by the spectral signature with the index *i*. Thus, it is possible to show that with such definition of these values the constraints on spectral coefficients mentioned in the optimization problem will be performed.

4. Experimental results of the spectral unmixing algorithm

To study the effeciency of the proposed spectral algorithm the synthesized hyperspectral images with 340 channels were used with a range of wavelengths from 0.8 to 2.5 microns with an increment size of 0.005 microns. Dimensions of the analyzed images were 64 × 64 pixels.

A raster mask for areas with the dimension of 512 × 512 was used as base map data.

Signatures from the spectral IGCP-264 Library [10] were used to generate test images. The coefficients of spectral signatures were set as stationary random fields with a biexponential correlation function. When forming images, the preliminary correction of coefficients was perfomed with regard to the constraint (3), and the normalization of coefficients was made in accordance with the constraint (2).
In order to receive the test (processed) image, first its detailed prototype was formed, i.e. a large-scale image with area masks, from which the test image was obtained by average values of the prototype hyperspectral readings as follows:

$$\bar{v}(n1,n2)=\frac{1}{T^2}\sum_{k_1,k_2=0}^{T-1}\bar{v}_p\left(n1\cdot T+k_1, n2\cdot T+k_2\right), \qquad (6)$$

where $\bar{v}_p$ is the hyperspectral reading of the detailed image-prototype, $\bar{v}$ is the reading of the processed image, $T\geq 2$ is the ratio of the linear dimensions of detailed and processed images.
To supplement the list of signatures the algorithm N-FINDR [9] with parameter EPS=10^{-6} was used.
An example of the test image and the area mask is shown in Fig. 1.
We shall denote a bright mask area as the Area-1, and a dark mask area as the Area-2. The Area-1 is formed by the mixture of signatures ALUNITE_AL705 and ILLITE_IL101, and the Area-2 is formed by the mixture of signatures SEPIOLITE_SEP3101, BUDDINGTONITE_NHB2301, HEMATITE_FE2602 from the IGCP-264 Library [10].

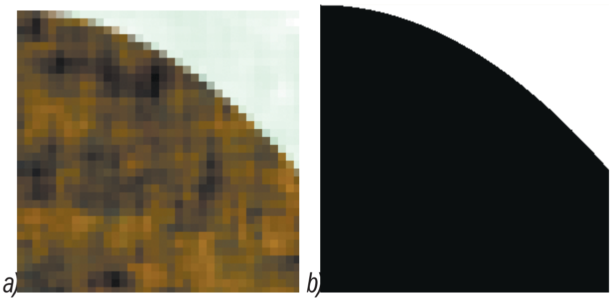

a) b)

Fig.1. a) the test image b) the area mask

The first part of the experiment was to investigate an error in estimating the spectral coefficients, when the signature set was fully defined. To calculate the error we used the following formula:

$$\xi=\frac{1}{|M|}\sum_{(n1,n2)\in M}\frac{1}{NS}\sum_{i=0}^{NS-1}\left(\lambda_i^b\left(n1,n2\right)-\lambda_i\left(n1,n2\right)\right)^2,$$

where M is the set of readings of the analyzed area, $|M|$ is the cardinality of M, $\lambda_i^b\left(n1,n2\right)$ are the basic factors for the signature with the number i in the list LS, $\lambda_i\left(n1,n2\right)$ is the evaluation of coefficients for the signature with the number i in the list LS. The set of entire pixels $\mathbf{V}$ or the set of edge pixels $\mathbf{V}^*$ were used in the capacity of the variety M.
Graph drawing of values for the mean-root-square error of coefficients is shown in Fig. 2. It is obvious that the unmixing coefficients are restored quite accurately within a wide range of a signal/noise ratio (additive uncorrelated noise with the normal distribution law with a zero mean and the given variance ratios are used in all experiments).

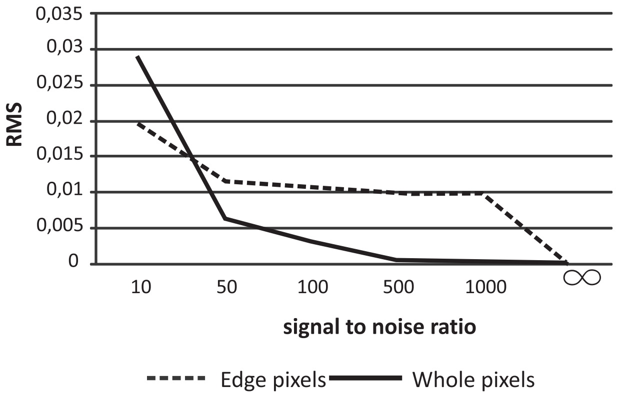


Fig. 2. The mean-root-square error for restoring the coefficients

The second part of the experiment was to study the representation quality of the original image readouts in the absence of signature information. The average absolute representation error of the hyperspectral pixel was applied as an indicator of the processing quality using the obtained coefficients:

$$\varepsilon=\frac{1}{N}\sum_{i=0}^{N-1}\bar{\varepsilon}_i ,$$

$$\bar{\varepsilon}_i=\frac{1}{|M|}\sum_{(n1,n2)\in M}\left|v_i\left(n1,n2\right)-\sum_{j=0}^{NSE-1}\lambda_j\left(n1,n2\right)s_{ji}\right|,$$

where N is the number of spectral components (channels), $\bar{\varepsilon}_i$ is the average absolute representation error in the reading of the hyperspectral image in i-m channel, M – is the set of readings of the analyzed area, $v_i\left(n1,n2\right)$ is the i -spectral component of the pixel $\left(n1,n2\right)$, s_{ji} is the i -spectral component of the j – signature from the list LSE.
Fig. 3 and Fig. 4 show the respective values ε for the considered test image for the known list of signatures in the absence of signature information (signatures extracted by N-FINDR are used).

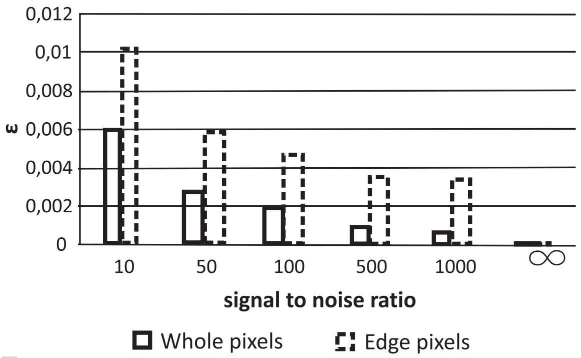


Fig. 3. The average absolute pixel representation error at different signal/noise ratios for the known list of signatures

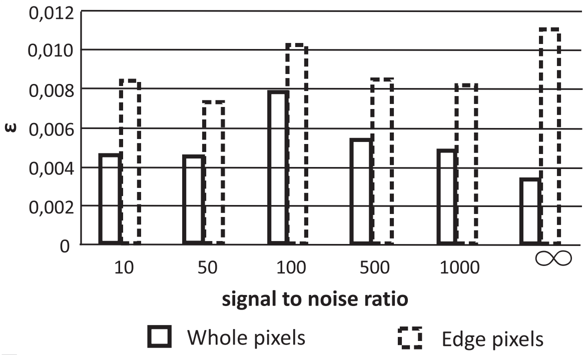


Fig. 4. The average absolute pixel representation error at different signal/noise ratios for the unknown list of signatures (signatures extracted by N-FINDR)

In general, it is seen that the representation error for the known set of signatures is smaller than that one needed to evaluate the set of signatures by means of N-FINDR method. For the complete list of signatures the error nature for entire and edge pixels in spectral channels is illustrated in Fig. 5. The graph drawing has been obtained when the signal/noise ratio was 100. Obviously, the smallest representation error refers to a smoothly varying signature areas. It should be generally noted that the method shows good noise immunity properties.

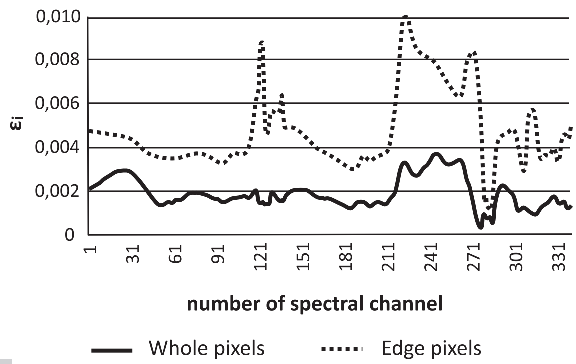


Fig. 5. The average pixel representation error at the signal/noise ratio of 100 and in the well-known signature list

5. Experimental results of the spectral selection algorithm

The efficiency of the spectral selection algorithm was studied on test images synthesized with the parameters and according to the scheme described above in the previous section. Fig. 6 shows the original image and the object mask.

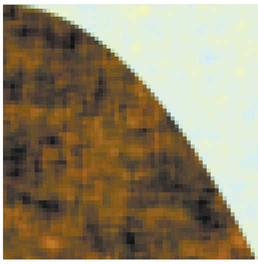
a)  *b)* 

Fig. 6. The test image and the objects mask for the spectral selection method

A white line in the figure corresponds to the object, the signature of which is extracted. The thickness of the object on the mask varies from 0.375 to 0.125 of the linear pixel size in the test image. The signature, which was not present in the background areas, corresponded to the map object.

The extracted signatures were compared with an object prototype signature KAOLINIT_KL502 from IGCP-264 Library by the root-mean-square deviation criterion. The results obtained in both two cases of the algorithm application – in the known list of signatures and in the unknown list of signatures (extracted my means of N-FINDR) – are shown in Fig. 7.

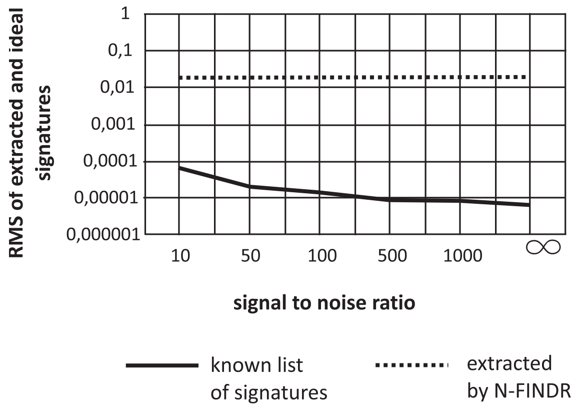


Fig. 7. The RMS graph drawing of extracted and prototyped signatures at different signal/noise ratios

Graph drawings in Fig. 7 show that the signature is restored quite accurately within the wide range of the signal/noise ratio (the worst error has the order of 0.01); moreover, the error value insignificantly changes within the tested noise range.

Conclusion

The paper has proposed two crucially new algorithms for the linear spectral mixture analysis in hyperspectral images, the specific nature of which is to use the image-registered areas, when solving the base map problems. For the first algorithm, the base map is used to clarify the spectral coefficients at the object edges introducing additional constraints in the problem description. For the second algorithm, the additional spatial information provided by the base map also specifies the corresponding formal criterion of the problem and allows to perform the spectral selection of small objects (i.e. the objects which has none of the entire spectral image reading) to extract their spectral signature. For both algorithms the set of applied spectral signatures may be either prespecified (with undetermined or predefined coefficients) or unknown and extracted in working on the algorithms. This fact makes it possible to assign the developed algorithms simultaneously to two classes of the hyperspectral analysis method – the methods of supervised (SLSMA) and unsupervised (ULSMA) linear spectral analysis.

The subpixel selection problem of the spectral signature in the proposed description (extraction of the signature for a small object, the dimensions of which can be smaller than the image readings) is new and, judging by the results of this work, it has been successfully solved using the base map. The existing methods of operation with hyperspectral images at the "sub-pixel" level may solve the problems of sub-pixel classification [1-3] and the target detection [1-3]; however, they do not remove the spectral signatures for small objects (the term "sub-pixel" in the problems of the spectral analysis mixture is understood as getting a part / a factor of the spectral coefficient of the entire hyperspectral image reading).

Further focus areas relate to the creation of numerically efficient methods for solving advanced problems of spectral unmixing, in which the traditional problem of linear spectral unmixing (2)-(4) turns out to be adjusted with additional constraints and/or changes in the objective function (similarly to the problems given in the second and the third sections of this paper).

Acknowledgments

This work was partially supported by:

– the RFBR grants (the Russian Foundation for Basic Researches) and the projects No. 12-07-00021-a, 13-01-12080-ofi-m, 13-07-97006-r_povoljie_a, 13-07-12103-ofi-m;

– the Fundamental Studies Program of the Presidium of the Russian Academy of Sciences "Fundamental Problems of Informatics and Information Technologies", project 2.12;

– the Ministry of Education and Science of the Russian Federation (within the framework of the Decree of the Government of the Russian Federation from 09.04.2010, No. 218: the contract No. 02.G36.31.0001 from 02.12.2013).

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