

# [13] Accuracy estimation of object tracking methods for identification of 2D-coordinates and velocities of mechanical systems based on digital photography data

Sklyarenko M.S.

Perm State University (National Research University)

## Abstract

Nowadays computer vision methods, in particular object tracking, are widely used by scientists and engineers. Primarily, object tracking is used in such applications as CCTV, crash tests, sports broadcasts and etc. Many methods can be adapted for non-contact optical measurements of coordinates and velocities of mechanical systems in physical experiments. Camera parameters influence on optical measurement accuracy is analyzed. We present comparative review of tracking algorithms and show how they can be used in mechanical experiments and analyze their accuracy in real mechanical experiments. Also we show our earlier developed methods adopted for the laboratory experiments which have subpixel accuracy and are robust to lightning differences. We present short review of tracking software. It is shown that object tracking techniques can be used for non-contact measurements and analysis of mechanical systems.

**Keywords:** MEASUREMENTS, MECHANICAL SYSTEMS, DIGITAL PHOTOGRAPHY, MOTION ANALYSIS, COMPUTER VISION, OBJECT TRACKING, HOUGH TRANSFORM, SEGMENT CROSS-CORRELATION METHOD.

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## Introduction

Experimental researches of complex mechanical systems are related to the necessity of applying several coordinate and velocity sensors. The number of sensors is proportional to the number of degrees of freedom. In many systems it is difficult to place sensors directly on moving subsystems. The complexity of sensors' placement and information collection considerably complicate real mechanical experiments. The use of non-contact optical measurement methods may greatly simplify the problem.

Mechanical motions can be directly observed within optical range. Many systems have rather high frequencies (100 Hz or more). As consequence, the main technological complexity in optical motion detection is the necessity of high-frequency photographing. Digital cameras capable to provide digital high-frequency photographing (up to 1 mln frames per second) have been recently appeared on the market (e.g. manufactured by Fastec, Photron, Vision Research). Development of recording equipment, on the one hand, and

object tracking methods (computer vision section), on the other hand, shall provide opportunities to wide use of optical measurements in experimental researches of complex kinematics systems.

Object tracking is an actively developing field of computer vision [1]. There is a great number of different object tracking techniques, however most of them are not applicable for real mechanical experiments. In major computer vision applications such as CCTV, estimation of traffic flow parameters [2], sports broadcasting (statistics on missile velocities, speed and total distance passed by athletes in team sports) [3], gesture recognition, etc., high accuracy is not usually required during coordinates detection. The methods are principally aimed at solving various related problems, i.e. object detection on a dynamic background in changing illumination conditions [4], increasing image quality [5] and on-line data processing.

On the one hand, real physical experiments specify strict requirements to accuracy of detection of objects' coor-

dinates; on the other hand, the experiments can provide better image recording and lighting constancy. The object may also be marked in different ways. This helps to avoid additional image processing and focus on the main problem, i.e. accurate detection of coordinates. In this context, researchers have begun to deal with adaptation of existing computer vision methods to real mechanical experiments, as well as with development of new techniques. Besides, some new tracking software and hardware have appeared on the market to solve tracking problems in laboratory experiments for a wide range of applications, i.e. sports biomechanics, computer animation, ballistics, etc.

Nowadays there is no appropriate information about opportunities and limitations of existing object tracking techniques for mechanical experiments data processing. The coordinates and velocities identification accuracy has been insufficiently investigated.

The objective of this paper is to provide comparative review and analysis of effectiveness of existing tracking methods for objects' coordinates and velocities identification in mechanical experiments.

We have hereby investigated the accuracy of objects' coordinates and velocities in model real mechanical experiments. The paper also describes specialized coordinate identification methods earlier developed by the author for mechanical experiments research applications, which have high accuracy and robustness to lighting changes.

The aforesaid 2D-coordinate identification methods can be extended to the case of 3D-coordinate identification by using several calibrated cameras (a computer stereo vision system) [6]. The methods have been tested on computer with 3GB RAM and Intel PentiumR 3.74 GHz CPU. For short, we will further name it a "test" computer.

### 1. Applicability of optical methods

When using photography for identification of coordinates and velocities of mechanical systems measurements accuracy mainly depends on digital camera parameters. The spatial resolution of most high-speed cameras (manufactured by Fastec, Photron, Vision Research) is over one megapixel. If we have to record relatively slow motions, we can use conventional (lowspeed) 60 fps videocameras with resolution of 8 megapixels (e.g., camera Sony FDR AX1 4K [7]).

Suppose that characteristic dimensions of objects and/or displacements are  $S$ , then at maximum resolution of 1 megapixel we can achieve the limiting coordinate measurement accuracy (at maximum lens magnification of the camera) up to  $10^{-3} S$ . The paper shall fur-

ther describe that by using specialized tracking techniques we can achieve subpixel accuracy (to several hundreds of pixel), so that the resolution can be increased up to  $10^{-3} S$ . When photographing objects 1m in size, we can achieve the limiting accuracy in coordinate measurement up to several hundreds of millimeters.

However, it should be noted that accuracy of coordinate measurements is not only referred to size of a camera matrix, but also to the finite exposure time. This error estimation technique is given below. Suppose  $\Delta T$  is the camera exposure time, and  $V$  is the characteristic velocity of object. Thus, while a shutter is open, the object will pass a distance equal to  $V\Delta T$ . The time interval between frames is  $1/f$ , where  $f$  is the frame frequency. During the time between two frames the object will cover the distance  $V/f$ . Since the exposure time is not equal to 0, the object will be blurred in the frame that will result to an error in coordinate measurements. The mean value of true value deviation from the measured value will amount to  $V\Delta T/2$  (a half distance passed by the object during the exposure time). The relative error in coordinate detection will be

$$V\Delta T/(2X) \quad (1)$$

where  $X$  is the true coordinate value.

As we can see, in order to reduce errors in coordinate measurements we need to decrease the exposure time. The use of short exposure times may require increase scene illumination.

This can be done by lighting source or a stroboscope synchronized with the camera.

The maximum two-frame displacement detection error will amount to  $V\Delta T$ , therefore the relative displacement detection error is  $\Delta T f$ . So, the relative error in identifying the velocity based on experimental data using the finite difference method will be equal to the relative displacement detection error, respectively. It should be noted that the exposure time and the frame frequency are not independent parameters; the exposure time may not be longer than the time interval between the frames, i.e.

$$\Delta T = \alpha / f \quad (2)$$

where  $\alpha$  – is a positive number that is less than 1. We identify that the relative velocity error is equal to  $\alpha$ . Substituting (2) to (1), we receive

$$V\alpha f/(2X) \quad (3)$$

The increased frame frequency results to the reduced coordinate detection error. We didn't consider accelerations in error estimations, i.e. the velocity was considered by convention to be constant, whereas the velocity is usually alternate in real systems. Therefore, in order to measure the velocity it is necessary to take such frame frequency at which velocity variations over the time interval between frames may be neglected, i.e.

$$A/(fV) \ll 1 \quad (4)$$

where  $A$  – is a characteristic acceleration.

## 2. Object tracking methods

The author offers to divide tracking methods into two categories, i.e. point and object-based methods. The first group of methods is based on the analysis of individual pixels or groups of pixels (rectangular and circular areas) on images obtained during the experiment. Methods of the second group are based on identification of complex geometry objects on images with further analysis of the objects themselves.

These methods require to use much more complicated algorithms of image processing, as pattern recognition methods [8]. The paper [1] has proposed a similar classification based on a divergent object representation, i.e. point methods; methods based on object representation by a set of simple geometric shapes (circles, ellipses, rectangles, etc.); methods based on tracking object's edges (contour) or silhouette.

**The principle of point methods** lies in the fact that the first frame we specifies a group of points which are tracked on image sequence. It is practically impossible to track the point in real experiments (there are many points with similar brightness and color on a single frame). Therefore, a neighborhood consisting of several points is allocated round each point. The simplest in implementation neighborhood form is a square window with an odd number of pixels centered at a reference point. The tracking is aimed at searching the most similar neighborhoods on a pair of images. In fact, the point tracking technique is reduced to a problem of optimization of the segments similarity function. For this purpose we can use various methods of optimization, i.e. stochastic optimization, gradient descent, etc. [9]. If take into consideration the finiteness of object motions during the time between two adjacent frames, it is often easier to implement a complete enumeration in the rectangular window which is certainly larger than the object characteristic displacement.

There are different approaches to searching the most resembling areas; one of the most reliable methods is the segment cross-correlation method. This method has been successfully proved in particle image velocimetry (PIV) [10] measurements of velocity fields in liquids which are based on supplementing light scattering particles into a flow and on tracking their displacements using digital photography data. Maximality of the following correlation index serves as a resembling criterion:

$$R = \left( \sum_{i=1}^N F_i S_i \right) / \sqrt{\sum_{i=1}^N F_i^2 S_i^2} \quad (5)$$

where  $F_i, S_i$  – are the brightness of corresponding pixels of segments on the first and second images,  $N$  – is the number of pixels on a rectangular segment. The segment displacement, which corresponds to the maximum correlation, identifies a displacement vector of the tracked object point. The equation (5) holds for gray-scale images, but it can easily be generalized in case of color images:

$$R = \left( \sum_{i=1}^{3N} F_i S_i \right) / \sqrt{\sum_{i=1}^{3N} F_i^2 S_i^2} \quad (6)$$

where by  $F_i, S_i$  we mean the brightness of RGB-components of each pixel. If the scene's lighting is relatively constant, we can use the simpler criterion [11]:

$$R = \sum_{i=1}^{3N} (F_i - S_i)^2 \quad (7)$$

It is obvious that in case of small neighborhoods the method would be less robust to image noise and lighting differences. Besides, the maximum accuracy of this method is 1 pixel.

Segment cross-correlation method was tested on the coordinate and velocity identification on image sequence of mathematical pendulum oscillations process. The pendulum represented a spherical plasticine body suspended on an elastic thread freely oscillating under gravity field. The motions were recorded by digital photocamera Canon IXUS 65 in 30 fps video mode.

The ball's center was identified as a tracked point. Fig. 1 shows the results (phase trajectories) of the coordinate and velocity identification by the segment cross-correlation method for different window sizes.

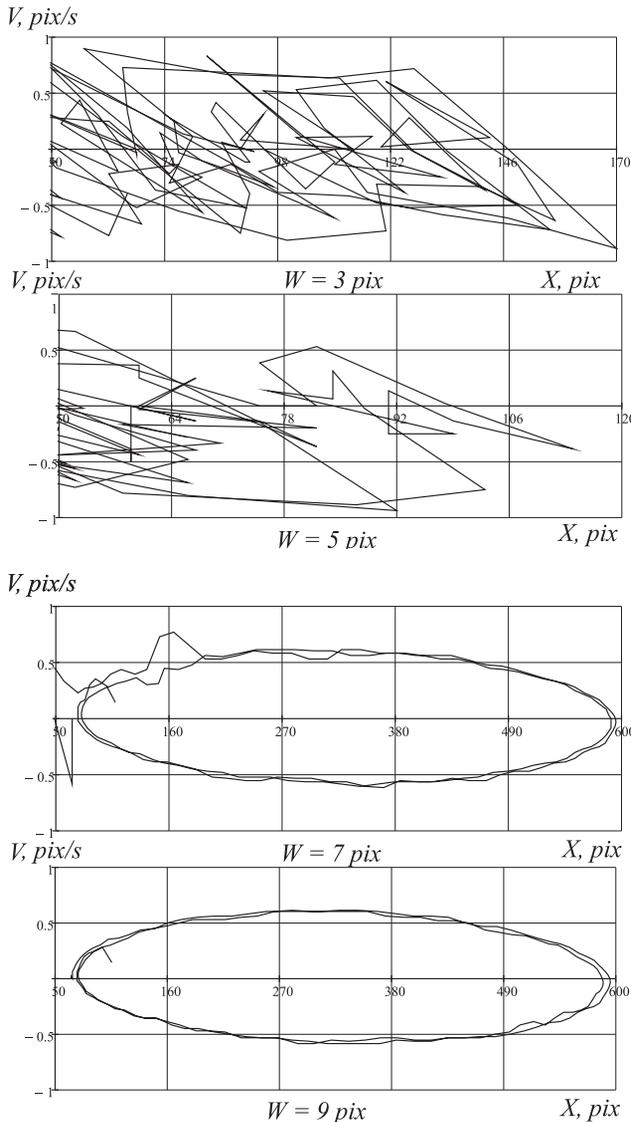


Fig. 1. Phase trajectories of the mathematical pendulum identified by the segment cross-correlation method for different sizes of the rectangular neighborhood  $W$

The velocity was determined by a two-point finite difference:

$$V_i = f(X_{i+1} - X_{i-1}) / 2 \quad (8)$$

where  $X$  – is the measured coordinate value,  $i$  – is the frame number,  $f$  – is the frequency of frames.

As is obvious (see fig. 1), it fails to identify the coordinate (the method “confuses” the points), and therefore to calculate the object velocity for small neighborhoods. The coordinate identification size increases with window size increase.

Let us see the following estimate of the dependence of the method accuracy on the window size. For this purpose the coordinate-and-time dependence was ap-

proximated using the least square method (we used the genfit function of MathCad 14.0 package) by the theoretical dependence (free small oscillations):

$$X(t) = A \cos(\omega t + \phi) + X_e \quad (9)$$

Error of the method was evaluated as approximation error:

$$\delta E = \sqrt{\frac{\sum_{i=1}^N (X_i - X(i))^2}{NA^2}} 100\% \quad (10)$$

where  $i$  – is the frame number,  $N$  – is the frame count,  $X_i$  – is the coordinate measurement value. We can similarly evaluate the velocity error. Fig. 2 shows a graph of errors depending on the segment size (in identification of approximation error, for explicitly invalid data from  $W = 3$ ,  $W = 5$ , the theoretical dependence specified for  $W = 9$  has been used). As is seen from the graphs, when exceeding a threshold  $W$  (in our example the threshold value was 7 pixels) the error practically stops falling, and there is no sense to use a larger window, since this requires additional computational costs.

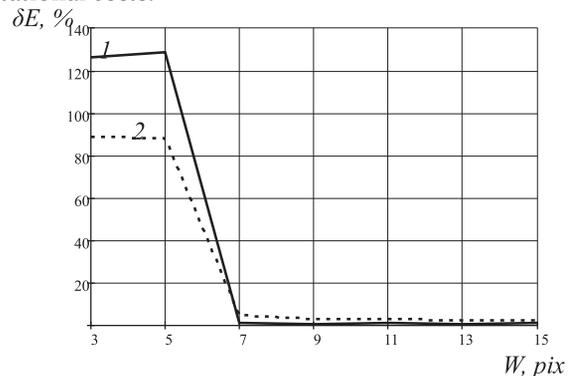


Fig. 2. The segment cross-correlation method: dependence of the relative error in the pendulum's coordinate and velocity measurements on the sizes of the square neighborhood  $W$ ; 1 – is the coordinate measurement error, 2 – is the velocity measurement error

The segment cross-correlation method has been implemented in language C++; the processing time of one frame,  $640 \times 480$  in size, for  $W = 9$  on the “test” computer was 2700 msec. Computational complexity of the algorithm is  $O(W^2 MN)$ , where  $M \times N$  – are the image dimensions.

Except the segment cross-correlation method there are also other techniques that are basically distinguished by a method of computation of the similarity criterion of neighborhood tracked points. The point methods are applied when there are differences in image brightness and color characteristics round the tracked point.

If we try to monitor the displacement of points of an evenly colored and illuminated object, the coordinate error may be equal to the object size, since the method will confuse the tracked point. It is also desirable that there are no points in the background which are similar to the object points; otherwise the error may exceed the size of the object several times.

As stated in the introduction, the existing tracking methods initially focus on tracking objects in real (not experimental) conditions. The number of objects, their size, shape, lighting conditions, and the nature of motions are generally considered to be unknown. Such methods are the most frequently used in CCTV systems in order to determine characteristics of the object motions in a protected zone [12].

Normally it is easy to provide constant uniform illumination in real experiments; you can also use unicoloured backgrounds placed beyond moving objects. Therefore, one can often succeed to easily detect the object from of the background. In that case, in contrast to the point methods the object is considered to be a certain group of points, i.e. a distributed tracer, whereas the object-based methods will take into consideration the object geometry (i.e. its size and shape). The algorithm of tracking in this case consists of several steps:

- 1) detection (selection) of tracers on a pair of frames;
- 2) identification of similar tracers;
- 3) detection (selection) and identification of pairs of characteristic points of the same tracers;
- 4) comparison of displacements of tracer's characteristic points with displacements of points of the measured object.

Let us consider the stage of tracers' detection. There are two approaches to select distributed tracers, i.e. silhouette and edge detection approaches [1]. In silhouette techniques an object silhouette is detected on the frame. The simplest method is a threshold detector. It is considered that pixel belongs to the silhouette if its brightness exceeds a certain threshold (if the object is brighter than the background, you can preliminarily take a negative image). Threshold and other silhouette detectors are very sensitive to uneven lighting along the scene and may result in significant silhouette distortions, and therefore the edge detection methods are preferred.

**The edge tracking methods** are based on detecting edges of sharp brightness differences on the object using the detector. There are many edge detection algorithms in the literature [5]. One of the most effective methods is Canny's algorithm [13].

As an example of the edge tracking we will consider the method developed by the author in 2006 [8].

We use Canny's algorithm in order to select tracers, and apply identification criterion for similar tracers detection (similarity or difference criterion). It is offered to characterize similarity by commonality of shape and dimensions which require quantitative characteristics for their description. Suppose the contour is initially represented by a simple ordered set of  $N$ -points with coordinates  $x_i, y_i, i = 1, 2, \dots, N$ , which correspond to the object contour. The paper [14] has offered for each edge to assign the 3D vector  $\vec{V}$ , for an each contour, the components of which are determined as follows:

$$V_1 = M_{20} + M_{02} \quad (11)$$

$$V_2 = (M_{20} - M_{02})^2 + 4M_{11}^2 \quad (12)$$

$$V_3 = (M_{30} - 3M_{12})^2 + (3M_{21} - M_{03})^2 \quad (13)$$

where  $M_{pq}$  - are central moments of the tracer,

$$M_{pq} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^p (y_i - \bar{y})^q \quad (14)$$

where  $\bar{x}, \bar{y}$  - are mean point coordinate values. This vector code is invariant to Euclidean coordinate transformations.

As a measure of inequality  $K$  we use the Euclidean distance between edges of the vectors  $\vec{U}$  and  $\vec{V}$  which correspond to the edges being compared. Components of the vectors have different orders in accordance with coordinates of the edge points, i.e. the first component means the second order, the second component means the forth order, and the third component means the sixth order. The lowest common multiple (LCM) is 12, therefore when calculating the distance you should use the following formula [8]:

$$K = \sqrt{(U_1 - V_1)^6 + |U_2 - V_2|^3 + (U_3 - V_3)^2} \quad (15)$$

We used this criterion in paper [11] which has shown rather good results.

After the edges corresponding to the object have been found in neighboring frames, the characteristic points have to be identified on them. The main characteristic point of the tracer is considered to be its "mass center." Coordinates of the edge "mass center" are identified as mean coordinate values based on the set of tracer's points. The displacement of the characteristic point is compared with the object displacement. If the object is rotated, then in order to describe its motions as motions of a rigid body, we have to know the displacement in several object points. In order to detect additional points we could apply the algorithm described in paper [15].

Because of using the distributed tracer, the edge tracking method enables to identify object coordinates with subpixel accuracy.

On the one hand, since the digital image recorded by a camera is discrete, it turns out that the accuracy in coordinate measurements may not exceed 1 pixel. On the other hand, considering that it's not the point but the edge with the distributed geometry which is identified in the photograph, the accuracy increases to fractions of pixel. Suppose a marker edge is shifted to the right at a distance of less than 1 pixel, e.g. 0.5 pix. Due to image discreteness some edge points will be inevitably displaced by one pixel (see Fig. 3) that will enable to identify the edge displacement.

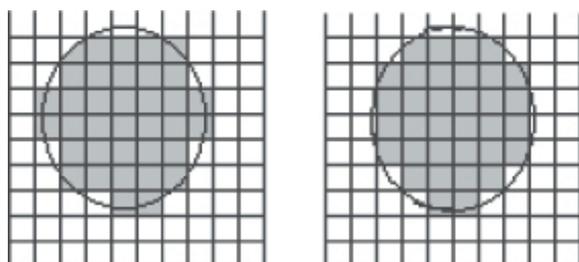


Fig. 3. About the subpixel accuracy: on the left is the initial edge position; on the right is the edge position at the next moment when it is shifted to the right by 0.5pix; pixels occupied with the marker more than a half are grey colored

The above described edge tracking method was applied to identify coordinates of the mathematical pendulum (see Fig. 4).

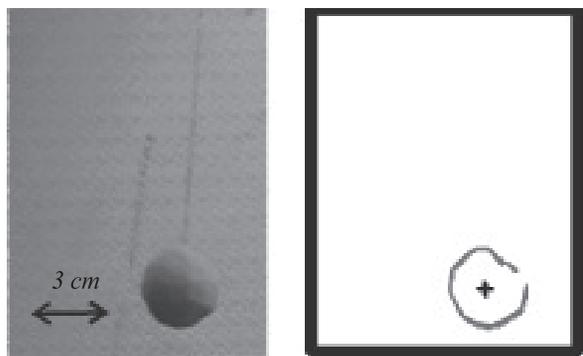


Fig. 4. The pendulum picture processed by the edge detection method: on the left is the original photo, on the right is the resulted identification of the marker edge using Canny's method [13]; a crossmark shows the edge center identified by averaging coordinates based on the set of edge points

It can be seen that because of low lighting difference in the right corner the edge discontinuity has appeared, however, it didn't corrupt identification of the pendulum's phase trajectory (see Fig. 5).

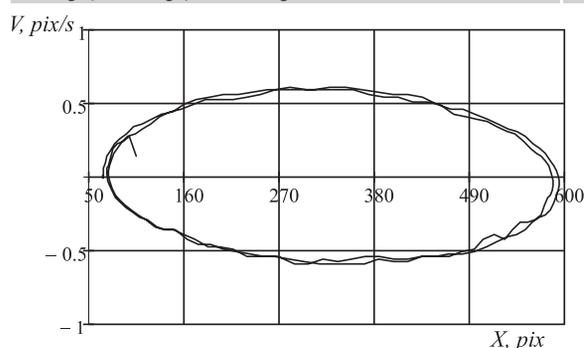


Fig. 5. Phase trajectory of free oscillations of the mathematical pendulum identified by the edge tracking method

Approximation errors of the coordinate-velocity-and-time dependence amounted to 1%, 3.6%, respectively. The edge detection method was implemented in language C++; the process time of one frame  $640 \times 480$  in size on the "test" computer was 140 msec. Computational complexity of the edge tracking method may be evaluated in accordance with the most resource-intensive step, i.e the edge detection using Canny's algorithm which has the order  $O(MN)$ , where  $M \times N$  – is the size of the image.

The edge tracking method is more stable if compared than the segment cross-correlation method and it can be applied even when the initial object is uniformly painted and lightened. It should be noted that if there are great differences in brightness on the object itself, it may result in considerable distortion of the object edge and the false identification of coordinates and velocities. The object tracking may be simplified by arranging on it contrast markers which have the specified size and shape. Round or ellipse-shaped markers (the white circle (ellipse) on the black background) are most often used. This significantly reduces the probability of the edge shape distortion. The purpose of tracking in this case is markers centers detection. The different approaches are used to identify the specified geometrical patterns in the image. One of the most popular and fast algorithms is the Hough transform [16,17].

**The Hough transform** is successfully applied in various computer vision applications, i.e. CCTV, defectoscopy [18], medicine [19], etc. The basic idea of the Hough transform is to detect, in the image with preliminarily enhanced edges (contours) (a gradient image), the analytical curves most accurately describing these edges.

In order to discuss algorithm of the Hough transform method we will consider the example of detection of straight lines in the image. In a polar coordinate system the equation of line is as follows:

$$X \cos \phi + Y \sin \phi = R \quad (16)$$

where  $R, \phi, -$  are unknown parameters [20]. After that the space sampling of parameters  $R, \phi$  is per-

formed, and a 2D array array (named then “accumulator”) is formed  $A_{R,\phi}$ . Then a so-called ‘voting procedure’ is fulfilled:

For each pixel of the gradient image, the brightness of which exceeds some threshold, the accumulator’s values shall be accumulated.

$$A_{R,\phi} = A_{R,\phi} + 1 \quad (17)$$

Such accumulation occurs only if pixel coordinates satisfy the above equation (16) within the sampling interval in parameters’ space.

The search of local maximums is carried out in the accumulator’s space. The maximums having polled more votes than the threshold number correspond to the desired straight lines. Similarly, the Hough transform may be used to search any other analytical curves, in particular, circles:

$$(X - X_c)^2 + (Y - Y_c)^2 = R^2 \quad (18)$$

where  $X_c, Y_c$  – are coordinates of the center,  $R$  – is a radius of circles. In parameters’ space  $(X_c, Y_c, R)$  we use an integer sampling interval; otherwise, the method would require more computational costs. Therefore, the tracking technique for circular markers based on the Hough transform has 1 pixel accuracy. Even when using a fractional sampling interval, the accuracy of the method will be limited with the sampling interval of the parameters  $X_c, Y_c$ .

The foregoing technique has been tested on identification of free damped oscillations of the physical pendulum representing a horizontally pivoted kernel executing small damped oscillations in gravitational field (for damping purpose a low end of the kernel was submerged in a vessel with water). The contrast marker (flat cardboard disc) has been placed on the kernel. The process shooting was carried out by the high-speed camera Basler WatchGuard A504kc at 400 fps (see Fig. 6). It required small exposure times setting; therefore additional DC lighting sources Rekam HaloLight 1000 were used.



Fig. 6. Picture of the unit for investigation of damped oscillations of the physical pendulum

The frame processing software has been implemented in C++ language using HoughCircles procedure of the OpenCV computer vision library [21]. This procedure used the modified Hough transform method [22].

The computational complexity of this method can be represented as follows

$$O(MN) + O_{\text{hough}} \quad (19)$$

The first summand is responsible for the stage of the edge detection, and the second – for the modified Hough transform. Using the modified general Hough transform method which has been discussed in details in the original paper by Prof. Davies E.R.[23], the computational complexity of the Hough transform may be reduced to  $O(E)$ , where  $E$  – is the number of edge points (points on the gradient image, the brightness of which is larger than the threshold). The number of points is generally proportional to linear dimensions of the image. Thus, the second summand may be neglected in (19), and the resulting computational complexity is  $O(MN)$ . The process time of one frame  $640 \times 480$  in size on the “test” computer was 130 msec.

Fig. 7 shows a dependency graph of the velocity-time relationship. It can be seen that velocities identification is failed, since the target object performs subpixel displacements during the time interval between two sequenced frames. The approximation error of the coordinate-time dependence of the theoretical curve amounted to 2.8%.

$V$ , pix/frame

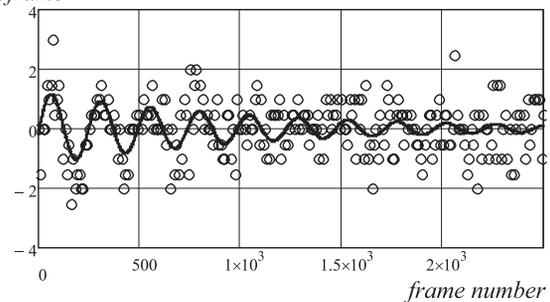


Fig. 7. Damped oscillations of the physical pendulum identified by the Hough transform method: the circles mean the experiment, and the full line means the approximation result

In this section we have considered the simplest Hough transform method for the circle tracking which has no subpixel accuracy. As of today, there are some extensions of the Hough transform method which have subpixel accuracy [24]. It should be noted that in order to search circles there are some alternative methods, for example, those based on the Radon transform which have subpixel accuracy [25].

### 3. The modified edge tracking method

As is shown in experiments with the physical pendulum, the coordinate measurement accuracy of one pixel may seem to be insufficient to detect velocities of mechanical objects in case when the object displacement between two frames is less than one pixel. Among all foregoing techniques only the edge tracking method proposed by the author has subpixel accuracy. In order to increase the method accuracy it is offered to fasten a circular- or ellipse-shaped marker on the object. However, the marker contour can be deformed because of lighting changes and insignificant blurring of the image due to fast motions of the object; a so-called 'fringe' can be formed in the edge, i.e. additional offset lines caused by sharp brightness differences in the marker's image. Besides, on one of the marker's edge sides some double lines may appear that would cause the increase of the number of points in this part of the boundary and, subsequently, the false shift of the marker's center identified by computing the average of coordinates. The line may also be discontinuous on one of the sides of the edge as a result of misspecified brightness thresholds in the Canny's edge detection method or because of flares falling to the marker's boundary (see Fig. 8a). All these distortions may result in infeasibility to identify velocities of objects using a numerical differentiation in real physical experiments. In order to reduce these errors the author have proposed in 2007 the modified method based on ellipse approximation of the marker's edge [8]. We offered the center of the approximated ellipse to be compared with the center of the marker. Fig. 8b shows the edge and centers of the marker detected by means of averaging and approximation. As we can see, the line twoness of the left boundary has caused the false shift of the center identified through averaging.

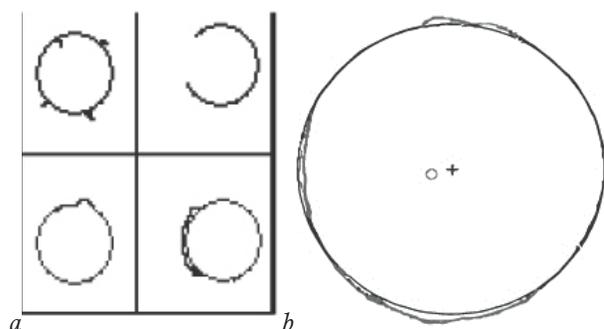


Fig. 8. Edges of the marker: a) possible distortions: from left to right top-down are the fringe, false discontinuity, shape distortion and doubled lines; b) detection of the marker's center: the small circle means the point set averaging, and the crossmark means the ellipse approximation

There are various ellipse or circle approximation techniques for sets of points in literature [26-28]. For ellipse approximation of the marker's edge we offer simple (in software implementation) method based on minimization of the approximation error using the stochastic iteration algorithm [29].

The modified edge tracking method has been tested on series of photographs of damped free oscillations (see Fig. 9).

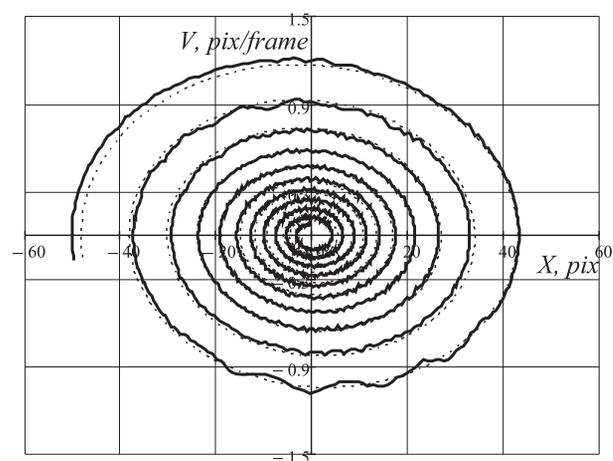


Fig. 9. Phase trajectory of damped oscillations for the physical pendulum identified by the modified edge tracking method: the full line means the experiment, and the dotted line means the approximation result; the approximation error is 1.7%

It should be recalled that we could not identify the pendulum velocity using the Hough transform method considering that the object could execute subpixel displacement during one frame.

Computational complexity of the modified edge tracking method may be represented as follows

$$O(MN) + O(EI) \quad (20)$$

where the first summand is responsible for the stage of the edge detection, and the second summand is responsible for the ellipse approximation of the marker's edge. In the second summand  $E$  means the number of edge points, and  $I$  means the number of iterations of the stochastic algorithm.

It should be noted that in many cases the second summand may considerably exceed the first one, since at least 3000 iterations are usually used [29], and the size of the edge is equal in order to the linear size of the image. Software for the modified edge tracking method has been implemented in language C++. The average process time of one frame  $640 \times 480$  in size on the "test" computer was 5100 msec (3000 iterations of the stochastic algorithm were used).

We have tested the considered method in series of experimental tasks, i.e. identification of non-linear [29] and forced oscillations of the pendulum, identification of forced oscillations of couple pendulums [30], motion analysis and identification of parameters of locomotion full-scale models [31]. The method has shown in all experiments practically total independence from lighting conditions and subpixel (up to several hundreds of pixel) accuracy in detection of the object's coordinates.

#### 4. Software review

Some new software and hardware complexes have recently appeared on the market to solve tracking tasks in physical experiment's conditions.

This section represents a summary review of the proposed basic solutions.

Most of the products are principally focused on tracking of actor's motions aimed at further creation of animated 3D models. Among these products we should focus on Vicon Motion Systems Ltd software [32]. The company produces software products such as Tracker, Blade, Nexus which enable online tracking of rigid bodies coordinates based on several cameras images processing. Software applications produced by this company are used in such industries as film industry, sports and defense biomechanics. These software products may be also used in other industries, particularly, in aerodynamics [33].

There are also products principally focused on scientific researches. SwissTrack solution [34] is a flexible OpenSource tracking software for multi-agent systems which is intended for robotic engineering and biomechanics. We should focus on commercial software TemaMotion [35] produced by Image Systems. This product allows us to carry out all kinds of tracking: point, edge, and silhouette tracking, and tracking of specified shaped objects which have subpixel accuracy. TemaMotion software is successfully used in such industries as automotive industry (in crash tests), sports biomechanics, defense industry (in firearms tests) [36], as well as in experimental physics, e.g. the paper [37] describes research results in measurement procedure of ring motion with the use of high-speed camera during electromagnetic expansion and Image Systems software. The analyzed products may be applied in physical experiments of mechanical systems with compound kinematics, i.e. models of machinery mechanisms, locomotions of biosimilar robots, complex oscillatory systems, etc.

We should specially focus on OpenCV – open-source library with the implementation of numerous

computer vision methods which is successfully used by researchers in creating and testing private algorithms including tracking systems. Particularly, the aforesaid SwissTrack solution uses OpenCV functions. If software is developed for Intel processors, you can use optimized implementations of some basic image processing methods from the commercial Intel IPP library [38]. OpenCv library provides opportunities to use the methods based on Intel IPP functions.

#### Conclusion

The table 1 provides a comparative analysis of the aforesaid techniques.

As you can see, the modified edge tracking method offered by the author shows the highest accuracy, but requires high computational costs. The ellipse approximation of the points set is computationally intensive; therefore, due attention should be given to alternative approximation techniques the computational complexity of which could be reduced up to  $O(E)$ . It is possible to apply the modified Hough transform method in the tasks not demanding subpixel accuracy, and the point methods, if it's impossible to allocate markers on the object. The methods discussed in this paper can be easily generalized to the case of studying systems with  $n$ -degrees of freedom.

For this purpose the tracking technique for several points is used in the point methods, and markers of different size, shape and color shall be used in the object-based methods.

In real mechanical experiments the object's displacement two frames does not often exceed some predetermined threshold. Therefore, computational complexity of many algorithms may be considerably reduced. It especially refers to the point methods; the search range for similar points is simply narrowed to them; and it is not always successful in the object-based methods, since the size of the processed segment on the image can't be less than the tracer (object form). In particular, in subpixel methods offered hereby by the author the accuracy is basically caused by a large number of edge points.

The fast development of computer vision methods and various recording devices (high-speed cameras and video-cameras of visible, IR and UV ranges) shall offer the challenge for their broad applications in scientific experiments for the analysis of complex kinematics systems.

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Table. Comparative analysis of the methods; legends:

*P* – the point method, *O* – the object-based method, *SCC* – the segment cross-correlation method, *ET* – the edge tracking method, *MH* – the modified Hough transform method, *MET* – the modified edge tracking method

Method	Method type	Resistance to lighting changes	Coordinate measurement accuracy (pix)	Computational complexity of image processing $M \times N$ in size:
SCC	P	Low	I	$O(W^2MN)$ , $W$ – neighborhood size
C	O	Medium	0.01	$O(MN)$
MH	O	Medium	I	$O(MN)$
MET	O	High	0.01	$O(MN) + O(EI)$ , $E$ – number of edge points, $I$ – number of approximation method iterations

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