

## NOVEL ZONING RULE FOR DESIGNING SQUARE FRESNEL ZONE PLATE

I.V. Minin, O.V. Minin

Novosibirsk State Technical University, Russia

A. Petosa, S. Thirakoune

Communications Research Centre Canada

### Abstract

An improved zoning rule is presented for designing a square Fresnel zone plate lens (FZPL). This new rule results in a higher gain when the FZPL is used as an antenna element or can enhance the focusing properties of the square FZPL when used to collimate an incident plane wave. The derivation of this improved zoning rule is presented along with simulated results for some typical cases.

### Introduction

Fresnel zone plate lenses (FZPLs) have found many applications in both optical and microwave fields either as lenses to collimate energy or as antennas for high-gain applications [1-4]. They offer the advantages of simple fabrication, thin profile and low cost, but are typically not as efficient compared to conventional shaped lenses or reflectors. In a recent publication, circular and square FZPLs were implemented to improve the sensitivity of antenna-coupled infrared detectors [1]. FZPLs were chosen since they could be fabricated using the same lithographic tools as used in fabricating the antenna-coupled detectors. Both circular and square FZPLs have been examined in terms of their focusing properties [1]. The design of circular FZPLs is well established and to focus an incident plane wave to a point located a distance  $F$  from the lens, the dimensions of each zone radius ( $r_i$ ) can be determined using:

$$r_i = \sqrt{i\lambda F + (i\lambda/2)^2} \quad (1)$$

where  $F$  is the focal length of the FZPL, and  $\lambda$  is the wavelength in the medium. For the case of square zones, approximations are required since the square boundaries do not coincide with the circular Fresnel zones. In earlier work, the dimensions of the square zones were determined by equating the area of each zone with that of the corresponding circular zone [1]. Thus the relation between half of the side of the square ( $L_i$ ) and the radius of the corresponding circular zone ( $r_i$ ) was [1]:

$$L_i = \frac{r_i \sqrt{\pi}}{2} = 0.8862 r_i \quad (2)$$

In other designs, the length of the each square zone is equated to the diameter of the corresponding circular zone [4, 5]:

$$L_i = r_i \quad (3)$$

This work proposes a third alternative design, which is derived in the following section. The performance of square FZPLs with these various zoning rules is then compared in the final section.

### The main part

To derive the new zoning rule, the geometry in FIG. 1 is considered, where the circle of radius  $R$  represents

the boundary of a particular Fresnel zone centered at point  $O$  in the X-Y plane and having a focal point located on the Z-axis at a distance  $F$  from the lens.

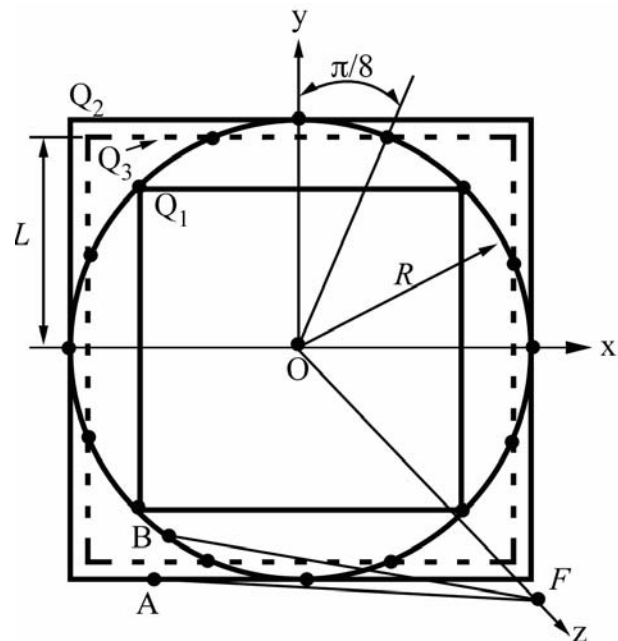


FIG. 1. Geometry of a single zone for deriving the improved zoning rule for a square FZPL

For any point  $B$  located on the circle, the following relation is precisely fulfilled within the geometric optics approximation for an axisymmetrical zonal plate (thus we shall consider that these boundaries are optimum for the appropriate axisymmetrical diffractive element):

$$|OB| + |BF| = |OF| + n\lambda/2 \quad (4)$$

The solid squares ( $Q_1$  and  $Q_2$ ) in FIG. 1 represent two possible choices for the square FZPL zone. If  $A$  represents a point on the square boundary and  $B$ , a point on the circular zone boundary, then one can define a maximum error  $\Delta$  between the difference of a set of rays from the lens aperture to the focal point as:

$$\Delta = (OA + (AF - ((OF + n\lambda/2) \neq 0) \neq 0) \neq 0) \neq 0 \quad (5)$$

The task of optimizing the square FZPL zones is one of minimizing this error  $\Delta$ . Using a simple geometrical interpretation, this error can be minimized by maximizing the number of common points between the square and circular zones. The two solid squares in FIG. 1 each

share only four points with the circular boundary. Choosing a square whose dimensions lie between the two solid squares (the dashed square labeled  $Q_3$  in FIG. 1) will increase the number of common points from four to eight and should thus reduce the error. The minimum error will occur when these eight points are equally spaced around the circular zone. The dimensions of the square zone can be related to the radius  $R$  of the circular zone for the various squares ( $Q_1$  to  $Q_3$ ) using simple trigonometry. If  $L$  is defined as half of the total length of the square zone, then

$$L = \begin{cases} R/\sqrt{2}, & \text{for } Q_1 \\ R, & \text{for } Q_2 \\ R \cos(\pi/8), & \text{for } Q_3 \end{cases} \quad (6)$$

The areas of these three square zones are compared to the area of the circular zone in Table I. It is important to notice, that this proposed technique for the definition of the width of the square zone does not depend on a type of the diffractive element (for example transformation of a flat or spherical wavefront set, or the focusing to a point or ring) and has a universal character.

Table I. Parameters for various zoning rules

Type of FZPL	Area of Zone	Ratio $S/S_0$
Circular	$S_0 = \pi R^2$	-
Equal Area Square <sup>1</sup>	$S = \pi R^2$	1.0
Interior Square ( $Q_1$ )	$S = 2R^2$	$2/\pi = 0.637$
Exterior Square ( $Q_2$ )	$S = 4R^2$	$4/\pi = 1.273$
Optimal Square ( $Q_3$ )	$S = 4R^2 \cos^2(\pi/8)$	$4 \cos^2(\pi/8)/\pi = 1.087$

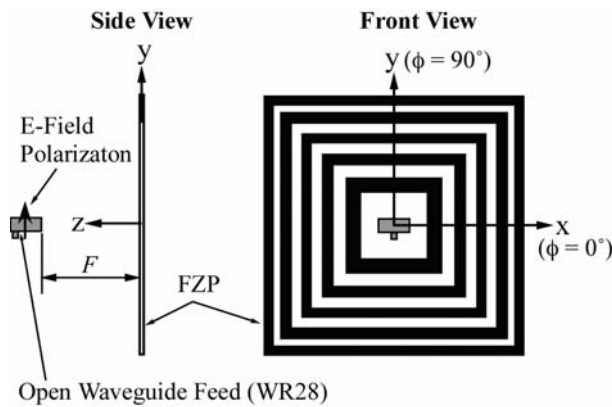


FIG. 2. Geometry of the square FZPL used as an antenna

**Discussion and conclusion**

To help verify the improved performance of this new zoning rule, several simulations were carried out using a commercially-available finite-difference time domain (FDTD) software [6]. The case presented here uses the square FZPL as an antenna, where the FZPL is illuminated by an open rectangular metal waveguide (WR28), as shown in FIG. 2, designed to operate at 30 GHz. The aperture of the waveguide is placed at the focal point of the FZPL, which is located at a distance  $F$  from the centre of the FZPL. Two lenses with 10 zones were analyzed: one with the  $L=R$  zoning rule ( $Q_2$  in FIG. 2) and one with the new zoning rule  $L=R\cos(\pi/8)$  ( $Q_3$  in FIG. 1) where the values of the circular zone radii were determined using (1) and  $F$  was chosen to be 37.5 mm. The simulated far-field patterns in the two principal planes ( $\phi=0^\circ$  and  $90^\circ$  as indicated in FIG. 2) are shown in FIG. 3 for these two lenses at 30 GHz. For this case, the new zoning rule improves the peak gain by 1.2

dB over the traditional zoning rule even for a binary amplitude-type of FZP. Simulations where the square FZP is used as a lens to collimate an incident plane wave show that the new zoning rule also enhances the focusing properties, by moving the focal point significantly closer to the lens aperture, compared to the traditional zoning rules.

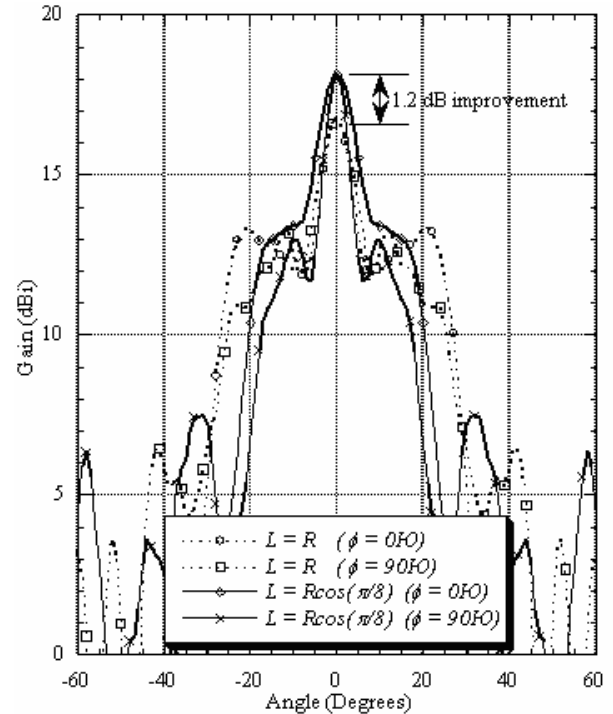


FIG. 3. Simulated gain of the FZPLs at 30 GHz

In conclusion, a new zoning rule for optimizing the performance of FZP lenses and antennas with square zones has been presented. An example using the square

FZP as an antenna designed at 30 GHz has shown an increase in gain when this new rule is applied. This new zoning rule may be used for square FZPs in microwave, millimeter and THz waves, optical and IR techniques for optimal design of diffractive elements with square zone topology, including antennas, lenses, and scanning diffractive axicon [7].

#### *References*

1. Gonzalez F.J., Alda J., Ilic B. and Boreman G. D. Applied Optics, 43, 33, (2004).
2. Hristov H.D. Fresnel Zones in wireless links, zone plate lenses and antennas, (Artech House, Boston, 2002).
3. Minin O.V. and Minin I.V. Diffractive optics of millimeter waves, (IOP Publisher, London, 2004).
4. Ojeda-Castaneda J. and Gomez-Reino C. (editors), Thomson, B.J. (general editor), Selected Papers on Zone Plates, (Washington: SPIE Opt. Engng Press, 1996).
5. Janicijevic L.J., Optics J. (Paris), 13, 199, (1982).
6. EMPIRE from IMST; www.empire.de
7. Minin V. and Minin O.V, Proceeding of the "Diffractive Optics 2005", 3-7 Sept., Warsaw, Poland, 2005, paper DO42.

# Novel zoning rule for designing square fresnel zone plate

*I.V. Minin<sup>1</sup>, O.V. Minin<sup>1</sup>, A. Petosa<sup>2</sup>, S. Thirakoune<sup>2</sup>*

*<sup>1</sup>Novosibirsk State Technical University, Russia,*

*<sup>2</sup>Communications Research Centre Canada*

## **Abstract**

An improved zoning rule is presented for designing a square Fresnel zone plate lens (FZPL). This new rule results in a higher gain when the FZPL is used as an antenna element or can enhance the focusing properties of the square FZPL when used to collimate an incident plane wave. The derivation of this improved zoning rule is presented along with simulated results for some typical cases.

*Keywords:* Square Fresnel Zone Plate, plane wave, zoning rule, plate lens.

*Citation:* Minin IV, Minin OV, Petosa A, Thirakoune S. Novel Zoning Rule for Designing Square Fresnel Zone Plate. *Computer Optics* 2006; 30: 62-64.

## **References**

- [1] Gonzalez FJ, Alda J, Ilic B, Boreman GD. Infrared antennas coupled to lithographic Fresnel zone plate lenses. *Applied Optics* 2004; 43(33): 6067-6073. DOI: 10.1364/AO.43.006067.
- [2] Hristov HD. *Fresnel Zones in wireless links, zone plate lenses and antennas*. Boston: Artech House; 2000. ISBN: 978-0-89006-849-6.
- [3] Minin OV, Minin IV. *Diffraction optics of millimeter waves*. London: CRC Press; 2004. ISBN: 978-0-7503-0907-3.
- [4] Ojeda-Castaneda J, Gomez-Reino C, eds. *Selected papers on zone plates*. Washington: SPIE Press; 1996. ISBN: 978-1-62841-004-4.
- [5] Janicijevic L. Diffraction characteristics of square zone plates. *J Opt* 1982; 13(4): 199-206. DOI: 10.1088/0150-536X/13/4/004.
- [6] EMPIRE. Source: [www.empire.de](http://www.empire.de)
- [7] Minin IV, Minin OV. Scanning properties of the diffractive "lens-plus-axicon" lens in THz. Proc "Diffraction Optics-2005" 2005: DO42.