A new SVD modification of the discrete-time filtering algorithm with finite-step autocorrelated measurement noise

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Abstract

The paper considers the discrete-time filtering problem in the class of the linear stochastic system models with finite-step autocorrelated measurement noise. A new SVD modification of the discrete-time filtering algorithm is proposed. It was proved that the new SVD algorithm is algebraically equivalent to the conventional one but has improved computational properties. The algorithm is implemented in MATLAB. The results of numerical experiments confirmed the working capacity and numerical efficiency of the new SVD algorithm when compared with the standard Kalman filter and the extended Kalman-type algorithm. In the case where the noise covariance matrix in the state equation is close to zero, the extended Kalman-type algorithm diverges and quickly loses its functionality due to a huge increase in estimation errors. At the same time, the proposed SVD modification remains workable and allows for calculating estimates of the state vector with high accuracy. The results obtained can be used to solve practical problems related to the processing and analysis of measurement data in areas such as target tracking, communication networks, signal and image processing.

Keywords: linear discrete-time stochastic system, autocorrelated measurement noise, discrete-time filtering algorithm, SVD factorization.

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Introduction

Nowadays, discrete-time filtering problems for stochastic system models with non-Gaussian noise, including estimating the state of dynamic systems with correlated and autocorrelated noises, have increased interest [1, 2]. This is explained by the fact that for such dynamic systems, the quality of parameter estimation can significantly exceed similar results obtained in the case of Gaussian noise [3], which is important for various practical applications [2, 4 – 6].

The solution to the discrete-time filtering problem for system models with autocorrelated noise in measurements has recently been obtained using a Kalman-type algorithm [7]. This algorithm has undoubted advantages in terms of the accuracy of calculated estimates compared to the conventional Kalman filter [3], especially when the finite step of the autocorrelated noise has a large value [8]. At the same time, the authors' analysis of publications in the field of research interest led to the conclusion that until now, for systems with autocorrelated measurement noise, the problem of numerically stable algorithm construction based on various covariance matrix factorization methods has not been stated and solved yet.

Among the currently existing multitude of numerically stable modifications of the conventional Kalman filter, SVD-based modifications developed for discrete-time stochastic systems with additive Gaussian noise have the most attractive computational characteristics (see, for example, [9 – 11]).

Thus, the aim of this work is to construct a new, numerically efficient SVD modification of the Extended Kalman-type discrete-time filtering algorithm for systems with finite-step autocorrelated measurement noise, based on the SVD factorization procedure.

1. Problem statement

Consider a discrete-time linear stochastic system of the form

$$x_k = Ax_{k-1} + w_k, \tag{1}$$
$$y_k = Cx_k + v_k \tag{2}$$

with finite-step autocorrelated measurement noise

$$v_k = \sum_{i=0}^{l} H_i \varsigma_{k-i}, k = 1, \dots, N, \tag{3}$$

where $x_k \in \mathbb{R}^n$ is a hidden state vector that needs to be estimated; $w_k \in \mathbb{R}^n$ is a process noise; $y_k \in \mathbb{R}^p$ is a measurement vector; $v_k \in \mathbb{R}^p$ is an autocorrelated measurement noise; $\varsigma_k \in \mathbb{R}^q$ is a random vector; $\varsigma_j = 0$ if $j < 0$; A, C and H_i are system matrices; initial state vector $x_0 \sim \mathcal{N}(\bar{x}_0, P_0)$.

Suppose the following: (i) w_k and ς_k are independent Gaussian sequences with zero means and covariance matrices Q and J, respectively; (ii) initial state vector x_0 is not correlated with w_k and ς_k; (iii) w_k is not correlated with ς_i, $i = 0,1,\cdots,k$.

Given (3), we can write

$$\mathrm{Cov}(v_k, v_{k-1}) = \mathrm{Cov}\left(\sum_{i=0}^{l} H_i \varsigma_{k-i}, \sum_{i=0}^{l} H_i \varsigma_{k-1-i}\right) = \sum_{i=0}^{l-1} H_{i+1} J H_i^T, \quad (4)$$

which means that v_k and v_{k-1} are correlated for $i = 2,3,\cdots,l$. Hence, the measurement noise v_k is finite-step (l-step) autocorrelated.

The problem of discrete-time filtering is to construct an algorithm for estimating the hidden state vector x_k using available measurements y_k $(k = 1,\dots,N)$. Since the measurement noise v_k is autocorrelated we cannot apply the optimal Kalman filter [3] which assumes the measurement noise is white. To solve the problem of discrete-time filtering, we will use the approach proposed in [7].

Consider an augmented vector $\varphi_k = (x_k^T, \varsigma_k^T, \varsigma_{k-1}^T, \dots, \varsigma_{k-l}^T)^T$. Using (1) – (3), let us construct the augmented system

$$\varphi_k = A_* \varphi_{k-1} + \varepsilon_k, \quad (5)$$
$$y_k = C_* \varphi_k, \quad (6)$$

where

$$A_* = \begin{pmatrix} A & 0_{n\times lq} & 0_{n\times q} \\ 0_{q\times n} & 0_{q\times lq} & 0_{q\times q} \\ 0_{lq\times n} & I_{lq} & 0_{lq\times q} \end{pmatrix}, \varepsilon_k = \begin{pmatrix} w_k \\ \varsigma_{k+1} \\ 0_{lq\times 1} \end{pmatrix}, C_* = (C\ H_0\ H_1\ \dots\ H_l),$$
$$E[\varepsilon_k \varepsilon_k^T] = Q_* = \mathrm{diag}\left(Q, J, 0_{lq\times lq}\right).$$

Noting that ε_k is white and uncorrelated with φ_k according to assumptions (i)–(iii), the Kalman filtering technique was used in [7] to obtain the optimal state estimate $\hat{x}_k$ and the corresponding error covariance matrix P_k.

In this paper, we propose a new SVD modification of the Extended Kalman-type filter (EKF) proposed previously in [7]. Our new SVD algorithm is based on the numerically efficient SVD factorization procedure.

2. The new SVD modification of the Extended Kalman-type filtering algorithm

It is well-known that singular value decomposition (SVD) is the most important matrix factorization technique because it produces numerically stable factors and is always applicable to any rectangular or square matrix, including those close to singular [12]. SVD is a highly popular and powerful approach for solving real-world problems in various fields of research. The SVD factorization has been actively applied in recent years in the fields of parameter identification and discrete-time filtering in stochastic dynamical systems. However, to the best of our knowledge, SVD-based discrete-time filtering methods for linear discrete-time stochastic systems with autocorrelated noise in measurements have not been constructed yet. The results obtained in this paper will help to eliminate this gap.

Let us consider the SVD factorization for a rectangular matrix $\mathcal{A}$ of rank r $\mathcal{A} = W\Sigma V^T$ where $\Sigma = \mathrm{diag}\{S_{r\times r}, 0\}$, $S = \mathrm{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r\}$, W and V are orthogonal (in general, unitary) matrices, and $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0$ are singular values of matrix $\mathcal{A}$.

For any symmetric positive definite matrix P, the result of SVD factorization has the form $P = \Theta_P D_P \Theta_P^T$, where Θ_P is an orthogonal matrix, D_P is a diagonal matrix. Next, let $D^{1/2}$ be the matrix square root of diagonal matrix D.

Now we can formulate our main result, which is a new SVD modification of the Kalman-type discrete-time filtering algorithm. Initial data for the algorithm: $\hat{\varphi}_0 = (\bar{x}_0^T, \varsigma_0^T, 0^T, \dots, 0^T)^T$; $\Pi_0^* = \mathrm{diag}(\Pi_0, J, I)$.

Algorithm 1. SVD-modification of the Extended Kalman-type filtering algorithm
Input: $\hat{\varphi}_0$, Π_0^*
// Initialization
1. $\mathrm{svd}(\Pi_0^*) \Rightarrow (\Theta_{\Pi_0^*}, D_{\Pi_0^*})$; $\mathrm{svd}(Q_*) \Rightarrow (\Theta_{Q_*}, D_{Q_*})$; $\Theta_{P_0^*} = \Theta_{\Pi_0^*}$; $D_{P_0^*} = D_{\Pi_0^*}$
2. **For** $k = 1,2,\dots,N$ **do**
// Time Update
3. $\hat{\varphi}_{k|k-1} = A_* \hat{\varphi}_{k-1}$
4. $\mathcal{A}_1 = \begin{bmatrix} D_{P_{k-1}^*}^{1/2} \Theta_{P_{k-1}^*}^T A_*^T \\ D_{Q_*}^{1/2} \Theta_{Q_*}^T \end{bmatrix}$; $\mathrm{svd}(\mathcal{A}_1) \Rightarrow (W_1, \Sigma_1, V_1^T)$
where $\Sigma_1 = \begin{bmatrix} D_{M_k^*}^{1/2} \\ 0 \end{bmatrix}, V_1 = \Theta_{M_k^*}$
// Measurement Update
5. $\mathcal{A}_2 = \left[D_{M_k^*}^{1/2} \Theta_{M_k^*}^T C_*^T\right]$; $\mathrm{svd}(\mathcal{A}_2) \Rightarrow (W_2, \Sigma_2, V_2^T)$
where $\Sigma_2 = D_{N_k}^{1/2}, V_2 = \Theta_{N_k}$

6. $\bar{R}_k = M_k^* C_*^T \Theta_{N_k}, R_k = \bar{R}_k D_{N_k}^{-1} \Theta_{N_k}^T$

7. $\mathcal{A}_3 = \left[D_{M_k^*}^{1/2} \Theta_{M_k^*}^T (I - R_k C_*)^T \right]$; $\text{svd}(\mathcal{A}_3) \Rightarrow (W_3, \Sigma_3, V_3^T)$

where $\Sigma_3 = D_{P_k^*}^{1/2}, V_3 = \Theta_{P_k^*}^T$

8. $\bar{e}_k = \Theta_{N_k}^T e_k$ where $e_k = y_k - C_* \widehat{\varphi}_{k|k-1}$

9. $\widehat{\varphi}_k = \widehat{\varphi}_{k|k-1} + \bar{R}_k D_{N_k}^{-1} \bar{e}_k$

// Computing the estimate $\hat{x}_k$ and matrix P_k

10. $\hat{x}_k = (I_n \ 0)\widehat{\varphi}_k$, $P_k = (I_n \ 0)\Theta_{P_k^*} D_{P_k^*} \Theta_{P_k^*}^T (I_n \ 0)^T$

11. **End**

Output: $\hat{x}_k, P_k$ $(k = 1,2,\dots,N)$.

Algorithm 1 gives us a way to calculate estimates of the state vector x_k and error covariance matrices P_k based on measurements containing autocorrelated noise v_k. It uses a numerically stable SVD factorization procedure, which allows for calculating SVD factors $\Theta_{P_k^*}$ and $D_{P_k^*}$ instead of the full covariance matrix P_k^*. Additionally, this new modification, unlike the Extended Kalman-type filter [7], avoids inversion of complete matrices, which is extremely important for maintaining numerical stability of the algorithm.

The main theoretical result of the paper is as follows.

Theorem 1. The new SVD modification (Algorithms 1) and the Extended Kalman-type filter (EKF) are algebraically equivalent.

The proof is based on the general form

$$\mathcal{A}^T \mathcal{A} = (W \Sigma V^T)^T (W \Sigma V^T) = V \Sigma^2 V^T.$$

of the SVD factorization.

3. Numerical examples

Let us consider a discrete-time linear stochastic system (1), (2), defined by matrices

$$A = \begin{bmatrix} 0.9 & 0.2 \\ -0.1 & -0.7 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}, J = 10, P_0 = \begin{pmatrix} 10 & 6 \\ 6 & 8 \end{pmatrix}. \quad (7)$$

Initial value of the state vector $x_0 = (2,3)^T$. We set the finite step size of the autocorrelated noise to $l = 10$.

Computer modeling was performed in MATLAB. We use the built-in svd() function to implement the SVD factorization in Algorithm 1. Matrices H_i $(i = 0,\dots,l)$ are generated automatically with random elements distributed uniformly over the interval $[0.1,0.9]$.

First, let us demonstrate the working capacity of our SVD-based algorithm. We examined the following three filtering algorithms: (i) the standard Kalman filter (KF); (ii) the Extended Kalman-type filter (EKF) [7]; (iii) the proposed SVD-modification (SVD-EKF), which is Algorithm 1.

To compare the estimation accuracy of these discrete-time filtering methods, the following set of numerical experiments is performed.

1. The discrete-time linear stochastic system (1) – (3) is simulated for 100 discrete time points to generate measurements with finite-step autocorrelated noise. The system matrices are determined by (7).
2. The estimation problem is solved by the three above-mentioned filtering methods, with the same measurement history, the same initial conditions, and the same noise covariances. It is worth noting that, for the standard Kalman filter, we calculate the measurement covariance matrix according to (4).

The results of numerical experiments are presented in tab. 1 which shows the root mean square errors (RMSE) of the estimates of the state vector x_k calculated using three algorithms: KF, EKF, and SVD-EKF.

Tab. 1. RMSE of the state vector estimates

Algorithm	RMSE		
	x_k^1	x_k^2	x_k
KF	7.8277	4.2881	8.9253
EKF	3.4945	1.6202	3.8518
SVD-EKF	3.4945	1.6202	3.8518

Having analyzed the data of tab. 1, we conclude that the EKF and SVD-EKF algorithms give the same result, which confirms the equivalence of the two forms, as well as the performance of the new SVD modification. At the same time, algorithms EKF and SVD-KF are superior to the standard Kalman filter in terms of estimation accuracy.

Next, we want to demonstrate the improved numerical properties of the proposed SVD modification. To do this, let us consider the same example, but with the matrix Q close to zero. This means that, in system (1) – (3), the state process noise has almost no influence on the state vector x_k.

We conducted a series of numerical experiments when $Q = \sigma^2 I_2$ where $\sigma = 10^{-8}$. Results of the state vector estimation are shown in tab. 2. Figs. 1 and 2 demonstrate the plots of errors of the state vector estimates.

Tab. 2. RMSE of the state vector estimates ($Q = \sigma^2 I_2$)

Algorithm	**RMSE**		
KF	0.0900	0.0233	0.0930
EKF	7.2249e+16	4.5514e+15	7.2392e+16
SVD-EKF	0.0233	0.0179	0.0294

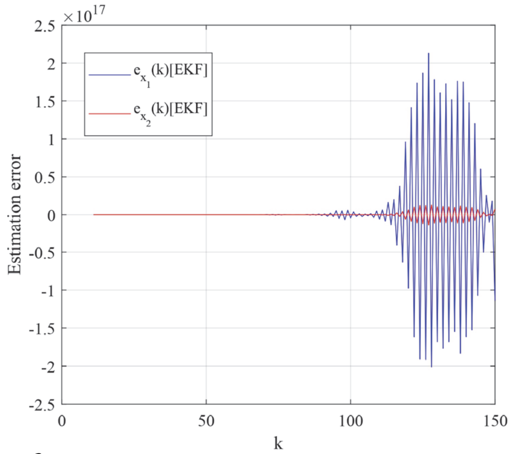

Fig. 1. Case $Q = \sigma^2 I_2$. Plots of the estimation errors calculated using algorithm EKF

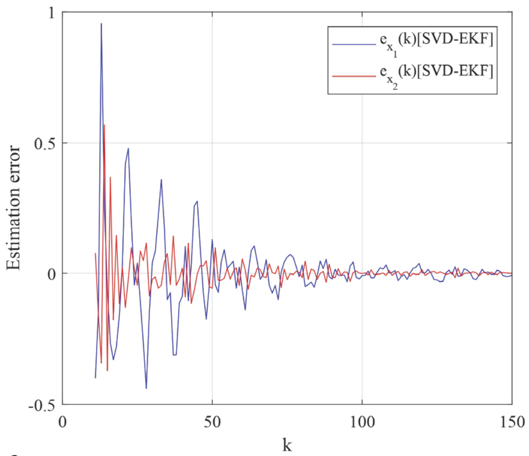

Fig. 2. Case $Q = \sigma^2 I_2$. Plots of the estimation errors calculated using algorithm SVD-EKF

Finally, a comparative analysis of the speed of algorithms was also conducted using the MATLAB timeit() function. The argument of this function is a descriptor of the algorithm whose speed needs to be checked. In this case, the functions implementing KF, EKF and SVD-EKF algorithms were compared. To perform a robust measurement result, timeit() calls the specified function multiple times and returns the median of the measurements. The speed measurement was performed at $N = 100$, where N is the number of discrete time instants. Tab. 3 shows the execution time in seconds.

Tab. 3. Execution time (s)

KF	**EKF**	**SVD-EKF**
0.000508	0.001020	0.016436

The results obtained allow us to draw the following conclusions. In the case where the noise covariance matrix Q in the state equation (1) is close to zero, algorithm EKF diverges and quickly loses its functionality due to a huge increase in the estimation errors. At the same time, the proposed SVD modification remains workable and allows for calculating estimates of the state vector x_k with high accuracy. It should also be noted that our algorithm has a clear advantage in terms of the accuracy of calculated estimates compared to the standard Kalman filter in the case of the autocorrelated measurement noise. As for the speed of the algorithms, SVD-EKF is about an order of magnitude slower than EKF, but in general, all three algorithms have high speed.

Thus, we have shown that, although Algorithms EKF and SVD-EKF are algebraically equivalent, their computational properties may depend on the stochastic system model and differ significantly in practical implementation on a computer.

Conclusion

The paper proposes a new SVD modification of the Kalman-type algorithm for discrete-time filtering of linear stochastic system models with finite-step autocorrelated measurement noise. The algorithm is based on the singular value decomposition of covariance matrices. It has the following advantages: a) it is numerically stable, because the numerically stable SVD factorization procedure is used to calculate covariance matrices; b) it allows for maintaining the symmetry of covariance matrices; c) it does not contain matrix inversion, which is extremely important for maintaining the numerical stability of the algorithm (only one inversion of a diagonal matrix is required); d) all eigenvalues of the error covariance matrices are automatically calculated at each step of the filtering algorithm and can be used to analyze the given model.

Numerical experiments confirmed the working capacity and numerical efficiency of our algorithm when compared with the standard Kalman filter and the extended Kalman-type algorithm for linear stochastic systems with autocorrelated noise in measurements. The disadvantages of SVD-EKF include a slower operation speed due to the use of a rather complex SVD factorization procedure. However, the difference in execution time compared to the standard algorithm EKF is not significant.

The results obtained can be used to solve practical problems related to the processing and analysis of measurement data in areas such as target tracking, communication networks, signal and image processing, etc.

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