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FOCUSATORS INTO THE LONGITUDINAL SEGMENT AND MULTI-FOCAL LENSES

1. Introduction

The phase optical elements mated to the spherical lens formate light field with longitudinally enhanced focal depth are referred to as the focusators into the segment [1]. Such optical elements can also be considered as some generalized axicons [2]. The computation technique for the phase function of such focusators that involves the ray-tracing equation in geometrical optics [1,2] does not take into consideration both the expansion of the ray pipes and their overlap that is the diffraction effects are neglected. This yields the discrepancy between the desired intensity distribution and that one which is formated actually by 30%-40% on the average [3].

Herein we propose the diffraction method for the computation of the phase function of the focusator into the longitudinal segment. The method makes it possible to compute the focusators which formate the longitudinal light segments with an arbitrary intensity distribution along the segments and provide the enhanced accuracy of such focusing, The algorithm is a modernization of the known iterative algorithm for the error minimization [4] that has been modernized by the adaptive procedure of the adjustment.

2. Computation algorithm

Let it be required to compute the radial phase function $\phi(r)$ of the optical element that formates light segment along the optical axis and limited by the points z_1 and z_2 with the arbitrary intensity distribution $I(z)$ on it, $z\in[z_1,z_2]$. In the Fresnel diffraction scalar approximation we can compute the light complex amplitude $F(\rho,z)$ at a distance of z from the optical element by

$$F(\rho,z)=\frac{k}{z}\exp\left(i\frac{k}{2z}\rho^2\right)\times \int_0^R \exp[i\phi(r)]\exp\left(i\frac{k}{2z}r^2\right)J_0\left(\frac{k}{z}r\rho\right)rdr \qquad (1)$$

where k is a light wavenumber, R is the focusator's radius, r, ρ are the radial variables in the plane of the focusator and the observer, respectively, $J_0(x)$ is the Bessel function of the zero order and of the first kind. The light complex amplitude on the optical axis at $\rho=0$ may be computed by formula:

$$F(0,z)=\frac{k}{z}\int_0^R \exp[i\phi(r)]\exp\left(i\frac{k}{2z}r^2\right)rdr \qquad (2)$$

After redesignation of variables

$$\xi=\frac{k}{z},\quad x=\frac{r^2}{2}$$

instead of (2) we can write

$$F(\xi)=\int_0^{x_0} \exp[i\phi(x)]\exp(ix\xi)dx\ ,\quad x_0=\frac{R^2}{2} \qquad (3)$$

As seen from (3) the complex light amplitude $\exp[i\phi(x)]$ behind the focusator is related to the light complex amplitude $F(\xi)$ on the optical axis through the Fourier transform.

Further, to compute the function $\phi(r)$ it is proposed to employ an iterative Gerchberg-Saxton algorithm [4] which solves the integral equation (3) by way of the sequential approximations. On the n-th iteration the light complex amplitude $f_n(x)$ which has been computed in the focusator's plane, should be replaced by $f_n'(x)$ function according to:

$$f_n'(x)=\begin{cases} f_n(x)\,|f_n(x)|^{-1}\ , & x\in[0,x_0] \\ 0\ , & x\notin[0,x_0] \end{cases} \qquad (4)$$

and the amplitude $F_n(\xi)$ computed on the optical axis should be replaced by the function $F_n'(\xi)$ by the rule:

$$F_n'(\xi)=\begin{cases} \sqrt{I(\xi)}\,F_n(\xi)\,|F_n(\xi)|^{-1}\ , & \xi\in[\xi_1,\xi_2] \\ 0\ , & \xi\notin[\xi_1,\xi_2] \end{cases} \qquad (5)$$

where $I(\xi)$ is the desired light intensity along z, $\xi_1=\frac{k}{z_2}$, $\xi_2=\frac{k}{z_1}$.

The phase $\phi(r)$ that has been computed on the basis of the formula (3) incorporates quadratic term describing a spherical lens. If one must compute the focusator into the longitudinal segment as the addition to the spherical lens, one should represent $\phi(r)$ as
so that the quadratic term be expressed in an

$$\phi(r)=\phi_0(r)-\frac{k}{2f}r^2 \qquad (6)$$

explicit form.

If the condition $2a=(z_2-z_1)<f$ takes place we can write instead of(2):

$$F(0,\Delta z)=\frac{k}{z}\int_0^R \exp\left[i\phi_0(r)\right]\exp\left(i\frac{k\Delta z}{2f^2}r^2\right)rdr \qquad (7)$$

where $\Delta z\in[-a,a]$, $f=\frac{z_1+z_2}{2}$ is the lens focal length. The relationship (7) with the help of the change of variables $\xi=k\Delta z f^{-2}$ and $x=\frac{r^2}{2}$ may be reduced to the Fourier transform similar to (3). Then, to compute the focusator phase function $\phi(r)$ we use the iterative procedures with the change (4) and (5).

3. Numerical results

The algorithm proposed was numerically evaluated in application to computing:

- the phase function of the focusator $\phi_0(r)$ into longitudinal segment with the constant intensity

$$I(\xi)=I_0\mathrm{rect}\left(\frac{f-\xi}{a}\right);$$

- the multi-focal lens with equal energy in each focus

$$I(\xi)=I_0\sum_{k=1}^{N}\delta(\xi-\xi_k)$$

Fig.1a shows the phase of the focusator into longitudinal line-segment with the constant intensity that was calculated during 55 iterations on the basis of formula (7). Fig.2a shows 2-D form of the same phase. The calculating conditions are the following: N=256 is a number of samplings on the focusator, R=4mm is the focusator radius, $k=10^4\,\mathrm{mm}^{-1}$, $z_1=380$mm, $z_2=420$mm, $f=(z_1+z_2)/2=400$mm is a lens focal length. In Fig.1b (curve 1) depicts calculated intensity distribution along optical axis from the focusator shown in Fig.1a. R.m.s. deviation of the calculated intensity from the required one (curve 2 , Fig.1b) amounted to 2%. Such high accuracy was a result of the adaptive adjustment procedure which was used on each iteration step [5,6]. After having been computing the phase (Fig.1a) was substituted into (1) and then the intensity distribution $I(\rho,z)$ was computed on a set of neighbouring planes ranging between

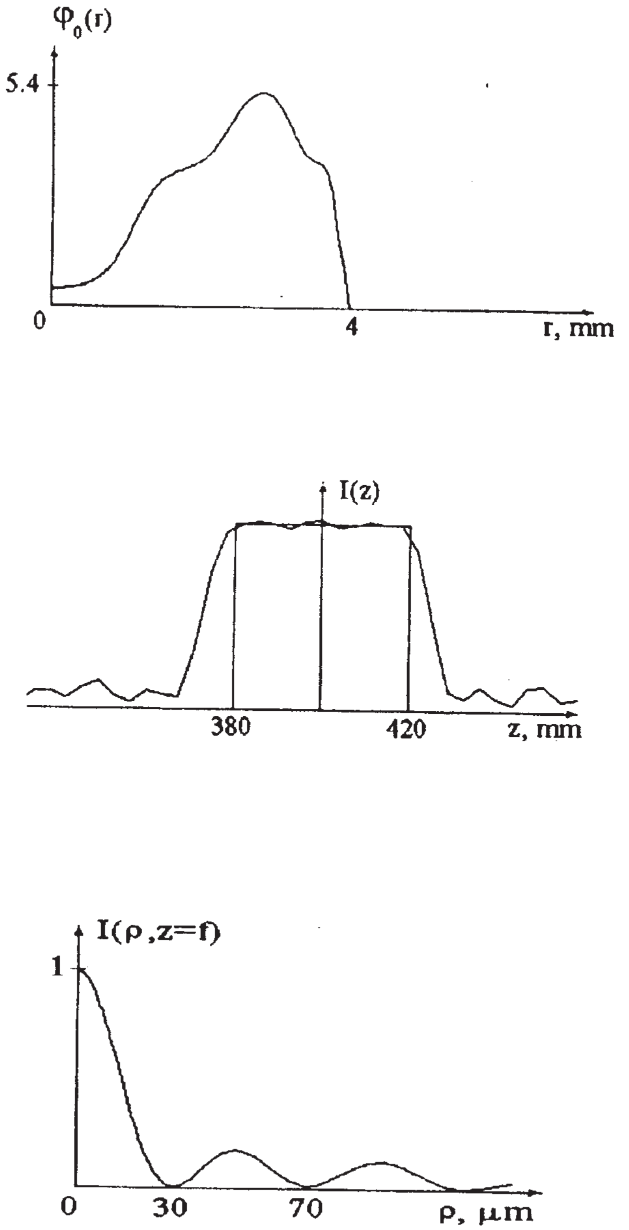

Fig.1 Computed during 55 iterations are:
a) the phase of the focusator into longitudinal segment with equal intensity, b) intensity distribution along an optical axis (curve 1), c) radial intensity distribution I(ρ,z).

$z_1=360$mm and $z_N=440$mm with step $\delta_z=2.5$mm. Fig.1c shows the radial intensity distribution for the focusator represented in Fig.1a in the plane of the geometrical focus with $f=400$mm. As seen from Fig.1c the lateral beam width was equal to 60μm. The beam width all the way from z_1 to z_2 was approximately the same. Fig.2b illustrates the form of the intensity distribution function $I(\rho,z)$ which has been computed for the focusator represented in Fig.2a. In that case p variations lie in the range: $-0.1\mathrm{mm}<\rho<0.1\mathrm{mm}$.

Fig.3a illustrates the phase of the focusator computed during 30 iterations which formates together with the lens five equidistant (spacing 20mm) focii with equal intensity in each of them. Resulted intensity distribution takes the form shown

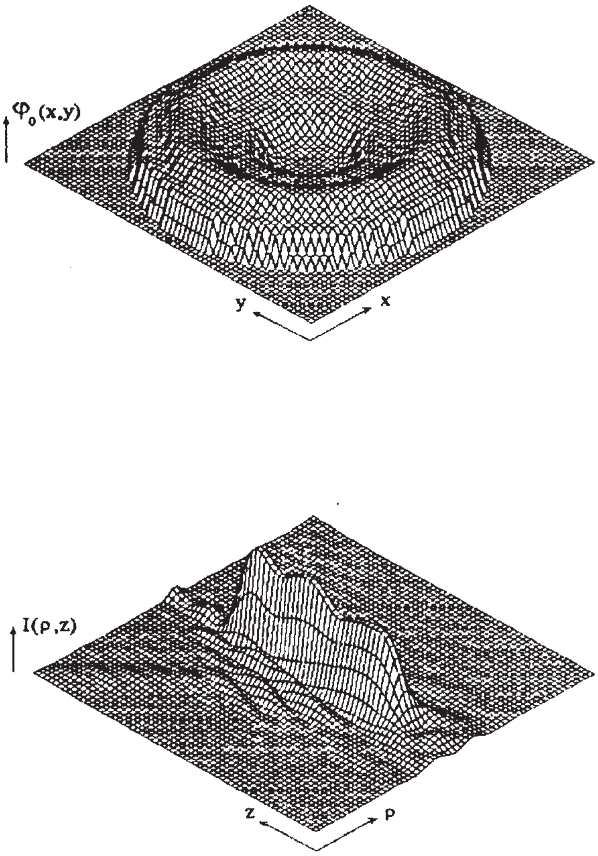

Fig.2
a) 2-D form of phase of the focusator into longitudinal segment,
b) 2-D intensity distribution I(ρ,z) in the area of focusing.

in Fig.3b. All five focii have equal intensities to the accuracy of 1%. Fig.4a illustrates 2-D form of the phase represented in Fig.3a. Fig.4b shows the light intensity distribution $I(\rho,z)$ from the multi-focal lens (fig.4a). The intensities were calculated for the planes lying between z_1=340mm and z_N=460mm with the step δ_z=2mm. The range of the variable variations was: -0.15mm<ρ<0.15mm. The numerical results presented above demonstrate the capabilities of proposed algorithm to synthesize optical elements - focusators into the longitudinal domains.

4. Conclusion

This paper is a continuation of the earlier works [5,6,7] where we have proposed the algorithm for diffractive calculation of the focusators into the lateral domains, namely, a circle, a square, a ring, etc. Herein we adapted the algorithm for calculating the focusators into the longitudinal domains: line-segment, a number of focii. The focusators into the longitudinal segments being the generalized axicons may find use in a series of practical tasks: light breakdown of gas [8], optical data recording and reading information from the disk [9], testing the surface [10] etc.

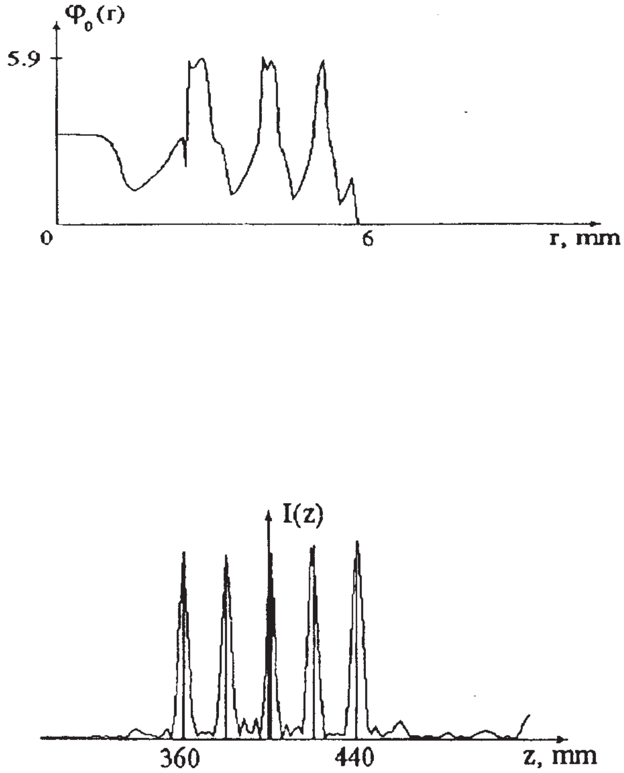

Fig.3 Computed during 30 iterations are:
a) the phase of the focusetor into five longitudinal focii
b) intensity distribution along the optical axis

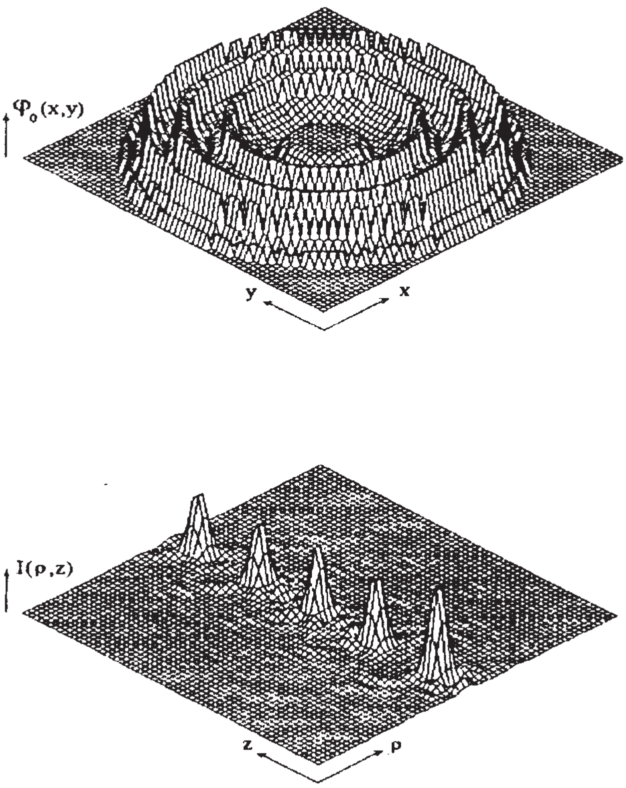

Fig.4
a) 2-D form of phase of the focusator into 5 focii,
b) 2-D intensity distribution I(ρ,z) in the area of focusing.

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