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ITERATIVE-PHASE METHOD FOR DIFFRACTIVELY LEVELLING THE GAUSS BEAM INTENSITY

1. Introduction

The demand for optical elements capable of levelling the Gaussian intensity distribution exists in such problems of optical data processing as laser-based superficial strengthening and laser projection printing. The diffraction redestribution of the light beam intensity can be performed by means of so called focusators of laser radiation [1,2]. Methods of geometrical optics make it possible to derive [2,3] analytical relationships for a focusator phase which is the smooth function of two variables. However, such an approach ignores diffraction effects that can result in low accurace in formation of the required focusator-produced intensity distribution. On the other hand, in computing a phase of kinoforms, an iterative algorithm [4-6] is employed, which takes into account diffraction effects taking place under the light propagation. Making use of the adaptive correction procedure [7,8] allows this algorithm to be successfully applied to compute the phase of focusators.

This paper deals with numerical comparison of performances of focusators from the Gaussian beam into the uniform intensity square computed by geometrical-optical with those computed by iterative methods.

2. Focusing from the Gaussian beam into a rectangle

As it has been shown [9], for the focusator that form the rectangular intensity-uniform distribution as

$$I(x,y)=\begin{cases}1, & |x|\le d_1,\ |y|\le d_2\\ 0, & |x|>d_1,\ |y|>d_2\end{cases}$$

from the plane beam with the Gaussian profile of the intensity distribution

$$I_0(u,v)=I_0\exp\left[-\frac{u^2+v^2}{\sigma^2}\right],$$

we can (given a lens located immediately adjacent to the focusator) find its phase function in the form of the sum

$$\phi(u,v)=\phi_1(u)+\phi_2(v)\ ,$$

with its terms satisfying the set of two simultaneous equations

$$\begin{cases}\dfrac{dx}{du}=\dfrac{I_0(u)}{I(x)}\\[2ex] x=u+fk^{-1}\dfrac{d\phi(u)}{du}\ ,\end{cases} \tag{1}$$

where f is the focal length of the lens in whose focal plane the light rectangle is formed, k is the wavenumber of light, (u,v) and (x,y) are the coordinates in the planes of the focusator and of the Fourier spectrum, respectively, $2d_1$, and $2d_2$ are the measures of the rectangle and σ is the parameter of the Gaussian beam.

The first equation in the system (1) sets the equality of the density of light energy of a $1D$ focusator to the density in the corresponding areas of the straight-light segment. The second equation in the system (1) describes the stationary points in the paraxial approximation.

The discrete variant of the solution of the system (1) can be written as follows

$$\phi_{mn}=\ln(10)\Big[M_x\Big\{6nN^{-1}\mathrm{erf}(6nN^{-1})+\pi^{-\frac{1}{2}}\exp(-36n^2N^{-2})\Big\}+M_y\Big\{6mN^{-1}\times\mathrm{erf}(6mN^{-1})+\pi^{-\frac{1}{2}}\exp(-36m^2N^{-2})\Big\}\Big]\ , \tag{2}$$

where $\mathrm{erf}(x)=2\pi^{-\frac{1}{2}}\int_0^x\exp(-t^2)dt$,

$m=0,\pm1,\pm2,\ldots,\pm N/2$, $n=0,\pm1,\pm2,\ldots,\pm N/2$, $M_x=\pi N_x(6\sqrt{\ln 10})^{-1}$, $M_y=\pi N_y(6\sqrt{\ln 10})^{-1}$, M_x and M_y are the number of the minimum diffraction spots that fall into the rectangle with the measures $N_x\times N_y$. In this case, the square aperture of the focusator measured $N\times N$ equals to the measure of the square, 6σ, and the Gaussian collimated beam that illuminates the focusator produces the light distribution as

$$I_{0mn}=\exp\left[-36N^{-2}(n^2+m^2)\right] \tag{3}$$

In section 4 we employ the phase (2) to numerically simulate the operation of a geometrical-optical focusator.

3. Adaptive-iterative computation of focusators

Following [7,8], let us briefly describe how the iterative algorithms can apply to computing the focusators as kinoforms. Let us proceed from the preset complex amplitude $A(u)$ of the illuminating beam and the required intensity distribution $I(x)$ in a focal plane of the lens. The complex amplitude $F(x)$ in a focal plane is related to the complex amplitude

$$f(u)=a(u)\exp[i\phi(u)]$$

immediately behind the focusator through the Fourier transform

$$F(x)=\int_{-b}^{b} f(u)\exp(ikxu/f)dx,$$

where $2b$ is the size of the focusator's aperture. The focusator's equation takes the form

$$|F(x)|^2=I(x) \tag{4}$$

To find iterative solution of Eq.(4) with respect to the phase $\phi(u)$, one should perform some preliminary estimate of the phase $\phi_0(u)$ followed by the computation of the complex light amplitude in a focal plane. In this case the complex amplitude $F_n(x)$ calculated in the n-th step of iterations is replaced by the function $F_n^0(x)$ according to the rule

$$F_n^0(x)=\begin{cases}\sqrt{I_n(x)}\,F_n(x)\,|F_n(x)|^{-1}, & |x|\le d\\ \alpha^{\frac{1}{2}}F_n(x) \quad , & |x|>d\end{cases}, \tag{5}$$

where $I_n(x)=(1+\alpha)I(x)-\alpha|F_n(x)|^2$, $I(x)$ is the required intensity distribution within the interval $[-d,d]$ of a focal plane, α is the parameter that controls the rate of convergence of the calculated intensity to the required one. For $\alpha=0$, the replacement (5) changes to the standard replacement in the Gerchberg-Saxton algorithm [5].

The amplitude of light in the plane of focusator $f_n(u)$ is calculated with the help of the inverse Fourier transform and is replaced by the function $f_n^0(x)$ according to the rule

$$f_n^0(x)=\begin{cases}A(u)f_n(u)\,|f_n(u)|^{-1} \;, & |u|\le b\\ 0 \;, & |u|>b \;.\end{cases} \tag{6}$$

In contrast to the algorithms of the conditional gradient [10], the α parameter is here introduced directly into the intensity function.

The rate of convergence of the intensity $|F_n(x)|^2$ to the required one $I(x)$ is checked by the root-mean-square deviation

$$\delta=\left[\frac{\int_{-d}^{d}[I(x)-|F_n(x)|^2]^2dx}{\int_{-d}^{d}I^2(x)dx}\right]^{1/2}. \tag{7}$$

Besides, a new parameter ϵ characterizing the energy efficiency of focusing is introduced:

$$\epsilon=\frac{\int_{-d}^{d}|F_n(x)|^2dx}{\int_{-\infty}^{\infty}|F_n(x)|^2dx}. \tag{8}$$

In the next section we apply the phase derived from Eqs. (4)-(6) to numerically simulating the operation of diffractive focusators.

4. Numerical results

We examine the focusator into a square that comprises 32×32 pixels and measures 10 minimum diffraction spots. For a focusator that focuses from the Gaussian collimated beam into a square, the phase deduced from Eq.(2) on the net of pixels 256×256 and taken to the modulus 2π represents the set of rings (lines of equal phase) changing to the lines of the square perimeter (Fig.1). Figure 2 illustrates the light intensity distribution in the lens focal plane calculated as the Fourier transform of the amplitude

Fig.1 Geometrical-optical phase og the focusator into a square.

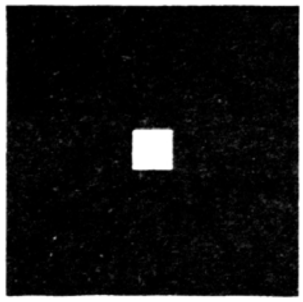

Fig.2 The intensity distribution obtained fron the geometrical-optical focusator in a lens focal plane.

$$f_{mn}=\sqrt{I_{0mn}e^{i\phi}mn}\;,$$

where I_{0mn} is taken from Eq. (3) and ϕ_{mn} from Eq. (2). The root-mean-square deviation of the obtained distribution from the uniform one amounted to 5% and the efficiency was 91.6%.

To calculate the focusator that focuses the Gaussian beam into the uniform intensity square, as an initial phase estimate we have chosen the geometrical-optical phase function described above (Fig.1). The subsequent iterative calculation has been conducted using three techniques. The first approach to the calculation of the phase based on the standard variant of the Gerchberg-Saxton algorithm with the replacement (5), for $\alpha=0$, yields, at first, the increase of the error δ in the first iteration steps and then it gives the slow decrease of the error (Fig.3a, curve 1). In this case during 10 iterations the error does not diminish below 13% though the efficiency ϵ grows up to 98.9% (Fig.3b, curve 1).

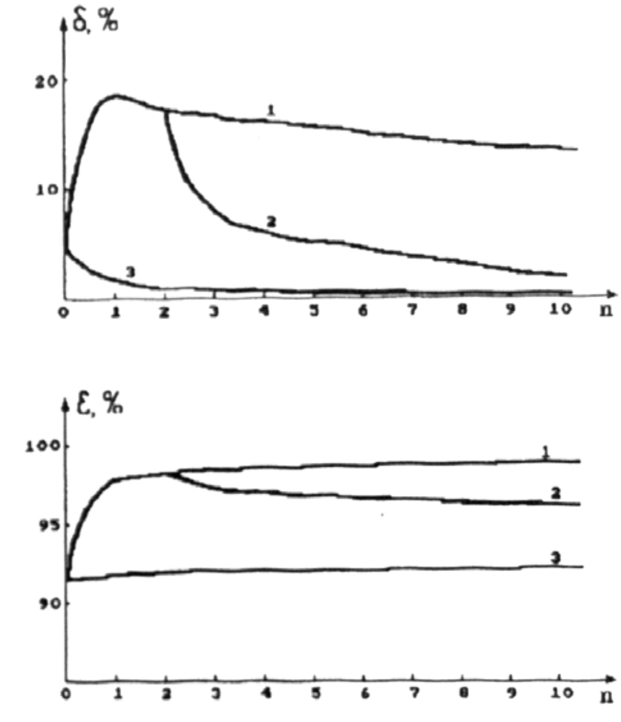

Fig.3 The root-mean-square deviation (a) at the energy efficiency (b) agains the number of iterations for different variants of an iterative algorithm: Gerchberg-Saxton (1), combined (2), and adaptive (3).

The second approach is combined: the first three iterations are conducted with the replacement (5) for $\alpha=0$, the remaining seven iterations for $\alpha=1$ (Fig.3a, curve 2). This method after 10 iterations results in formation of the square characterized by the uniform intensity, with the error of 2.4% and the efficiency of 96.4% (Fig.3b, curve 2). This way is seen to be more effective as compared with the first approach, since it yields the decrease of the error from 13% to 2% (by 6-fold) without essential decrease in the efficiency.

The third method is purely adaptive, this means that the replacement (5), with $\alpha=1$, is performed in each step of iteration (Fig.3a, curve 3). As one can see in this case, the error decreases monotonously and during 10 iterations it becomes equal to 0.1%, but the efficiency falls to 92.2% (Fig.3b, curve 3). Figure 4 illustrates the phase of the focusator to the modulus 2π that has been calculated during 10 iterations on the basis of the third approach from the geometrical-optical phase (2). First, one can see that the use of the iterative algorithm does not result in the essential change of the initial phase. The pronounced changes of the phase (Fig.1) take place only on the edges of the focusator (Fig.4). Second, we can draw the conclusion that the adaptive iterative algorithm based on the replacement (5) for $\alpha=1$ makes it possible to improve the geometrical-optical phase in such a manner that the error in the formation of a light square reduces more than by an order, with the efficiency almost unchanged.

Fig.4 The phase of the focusator into a square obtained after 10 iterations from the initial geometro-optical phase.

Fig.5 The radius-random phase chosen as the initial estimate for iterations.

For the aim of comparison, there was conducted another numerical simulation where we made use a radius-random phase function (Fig.5) as the initial approximation. The phase shown in Fig.6 was calculated during 10 iterations. The intensity distribution formed by the focusator characterized by such a phase is presented in Fig.7. The error and the efficiency were equal $\delta=6.4\%$ and $\epsilon=80.8\%$, respectively. Further iterations did not produce the essential change in these values. Data given above show that the iterative algorithm that begins with the random phase estimate, leads to the non-regular structure of the focusator's zones and results in the accuracy and efficiency which are somewhat less than those for a geometrical-optical focusator.

Fig.6 The phase of the focusator into a square obtained after 10 iterations from the initial random phase.

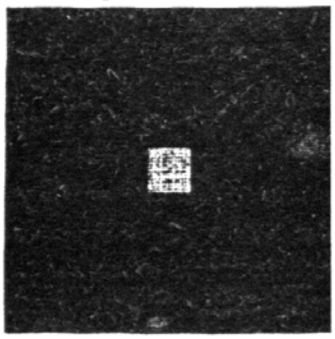

Fig.7 The intensity distribution in a lens focus obtained from the focusator with a phase shown in Fig.6.

The table summarizes all the focusators discussed in this paper. One can see the advantages in terms of the uniformity, the efficiency, and the regular phase structure that we achieve when using physically warranted initial approximation in the form of the geometrical-optical phase (compare rows 1-4 with row 5). The comparison between rows 1 and 2 in the table shows that the Gerchberg--Saxton method yields the increase of 7% in the energy efficiency of the geometrical-optical approximation (row 1) but produces considerable non--

Table

Type of a focusator and a phase	The ene-rgy eff-ici-ency ϵ, %	The relative r.m.s. error δ, %
Geometrical-optical phase	91.6	5.0
10 iterations of the Gerchberg-Saxton algorithm on the geometrical-optical phase	98.9	13.0
10 iterations of the adaptive-iterative algorithm on the geometrical-optical phase	92.2	0.1
10 combined iterations on the geometrical-optical phase	96.4	2.4
10 iterations on the radius-random phase	80.8	6.4

uniformity of the intensity over the square (13% instead of 5%).

On the contrary, the adaptive method ensures the high degree of the intensity uniformity (0.1%) but it practically does not improve the efficiency as compared with the geometrical-optical approximation (row 3). The combined method enables us to obtain rather high efficiency in combination with the quite satisfactory intensity uniformity (row 4 in the table). Row 5 shows that not knowing the geometrical-optical solution but based upon the random initial phase in the estimate of the focusator's phase, and using the iterative method we can achieve suitable results in terms of the accuracy and the efficiency in the formation of the required intensity distribution.

5. Conclusions

In the present work the authors have numerically shown that an optimal approach to a problem of computation of phase optical elements focusing the laser light into the small areas of the spatial spectrum plane consists in the solving of the inverse geometrical-optical task and finding of the phase function which is then chosen as the initial approximation for the adaptive-iterative procedure of obtaining of tn final phase. In this case the iterative procedure of the correction for the initial phase with the regular zone structure does not result in its essential change.

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